The QCD axion, precisely.

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The Abdus Salam International Centre for Theoretical Physics (ICTP)

based on:

1511.02867 : JHEP 1601 (2016) 034

w/ G. Grilli di Cortona, E. Hardy, J. Pardo Vega

1512.06746 :

w/ C.Bonati, M.D'Elia, M.Mariti, G.Martinelli, M.Mesiti, F.Negro, F.Sanfilippo

the problem

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad\qquad \theta = \theta_0 + \arg\det M_q$$

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$10^{-2} \div 10^{-3}$ $e\,\mathrm{GeV}^{-1}$

$$\Rightarrow \quad \theta \lesssim 10^{-10}$$

the axion solution

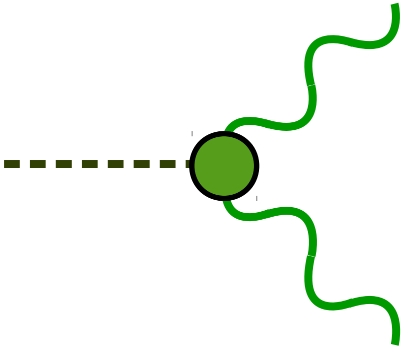
the QCD axion: *what it is*

$$a \rightarrow a + \delta_{\text{PQ-symmetry}}$$

the QCD axion: *what it is*

Weinberg, Wilczek '78

$$a \to a + \delta_{\text{PQ-symmetry}}$$



$$\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

the QCD axion: *how it couples*

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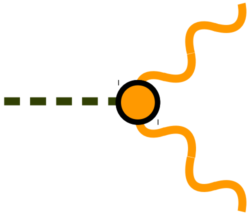
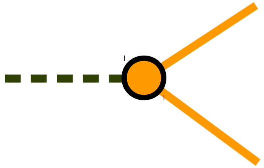
$$\frac{1}{4}\,a\,g^0_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}$$



$$c^0_q\bar{q}\gamma^\mu\gamma_5 q\,\frac{\partial_\mu a}{2f_a}$$



the QCD axion: *how it couples*

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$$c_q^0\bar{q}\gamma^\mu\gamma_5 q\,\frac{\partial_\mu a}{2f_a}$$

$q \to e^{i\gamma_5 a/f_a} q$

$$\bar{q}_L M_q e^{iQ_a a/f_a} q_R + h.c.$$

the QCD axion: *how it solves the problem*

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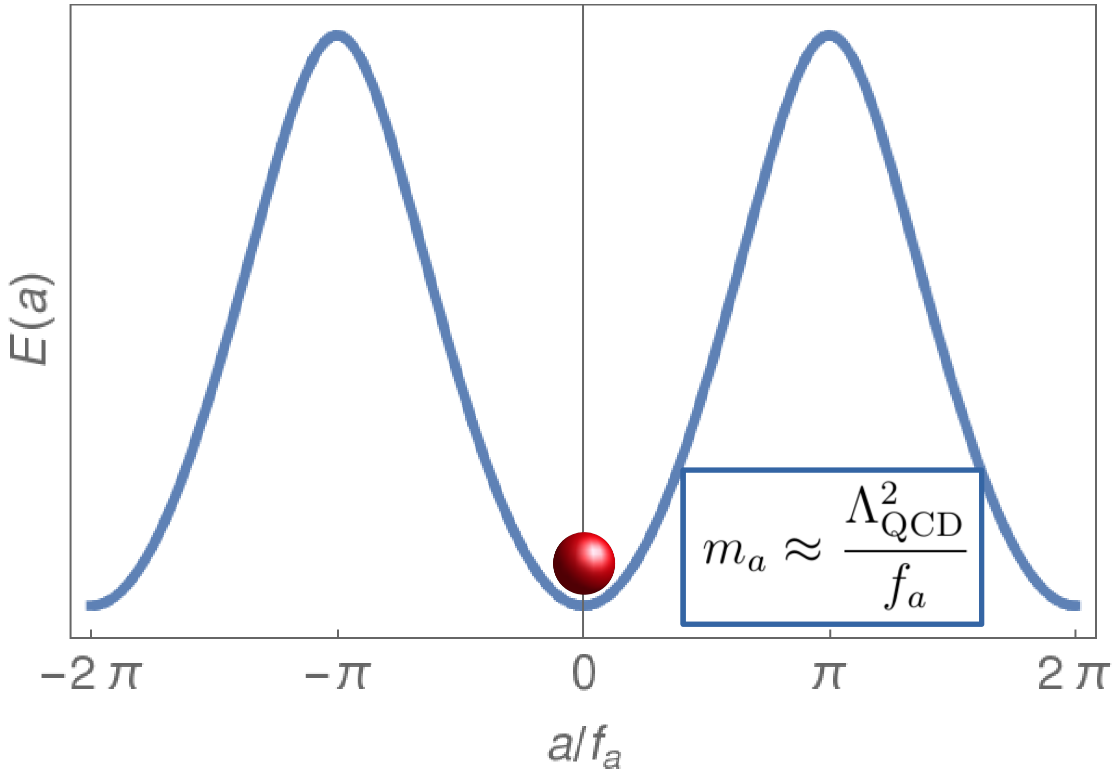
$$\begin{aligned} e^{-V_4 E(\theta)} &= \int \delta[\phi] e^{-S_0 + i\theta Q} \\ &= \left| \int \delta[\phi] e^{-S_0 + i\theta Q} \right| \\ &\leq \int \delta[\phi] \left| e^{-S_0 + i\theta Q} \right| \\ &= e^{-V_4 E(0)} \end{aligned}$$

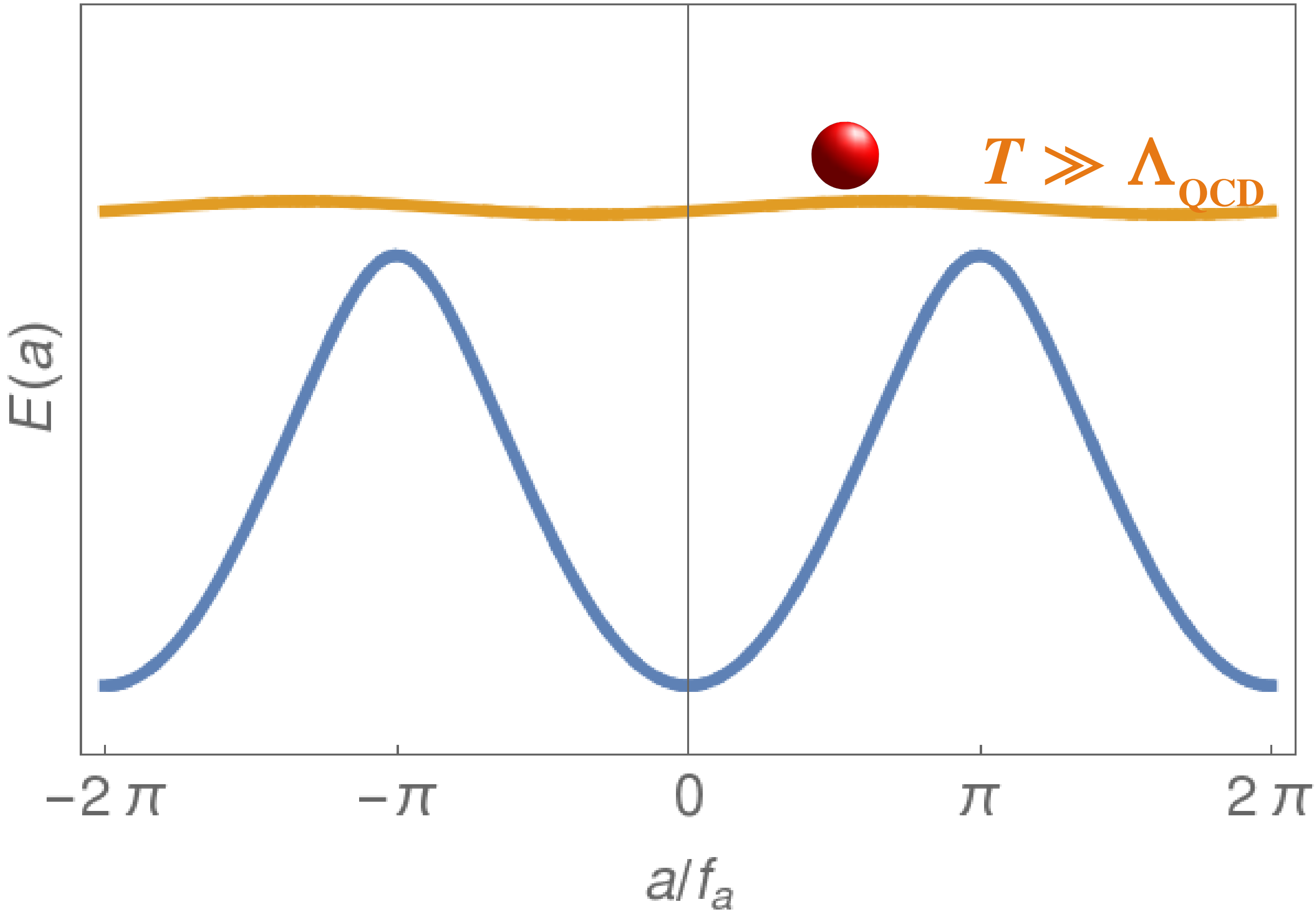
Vafa Witten '84



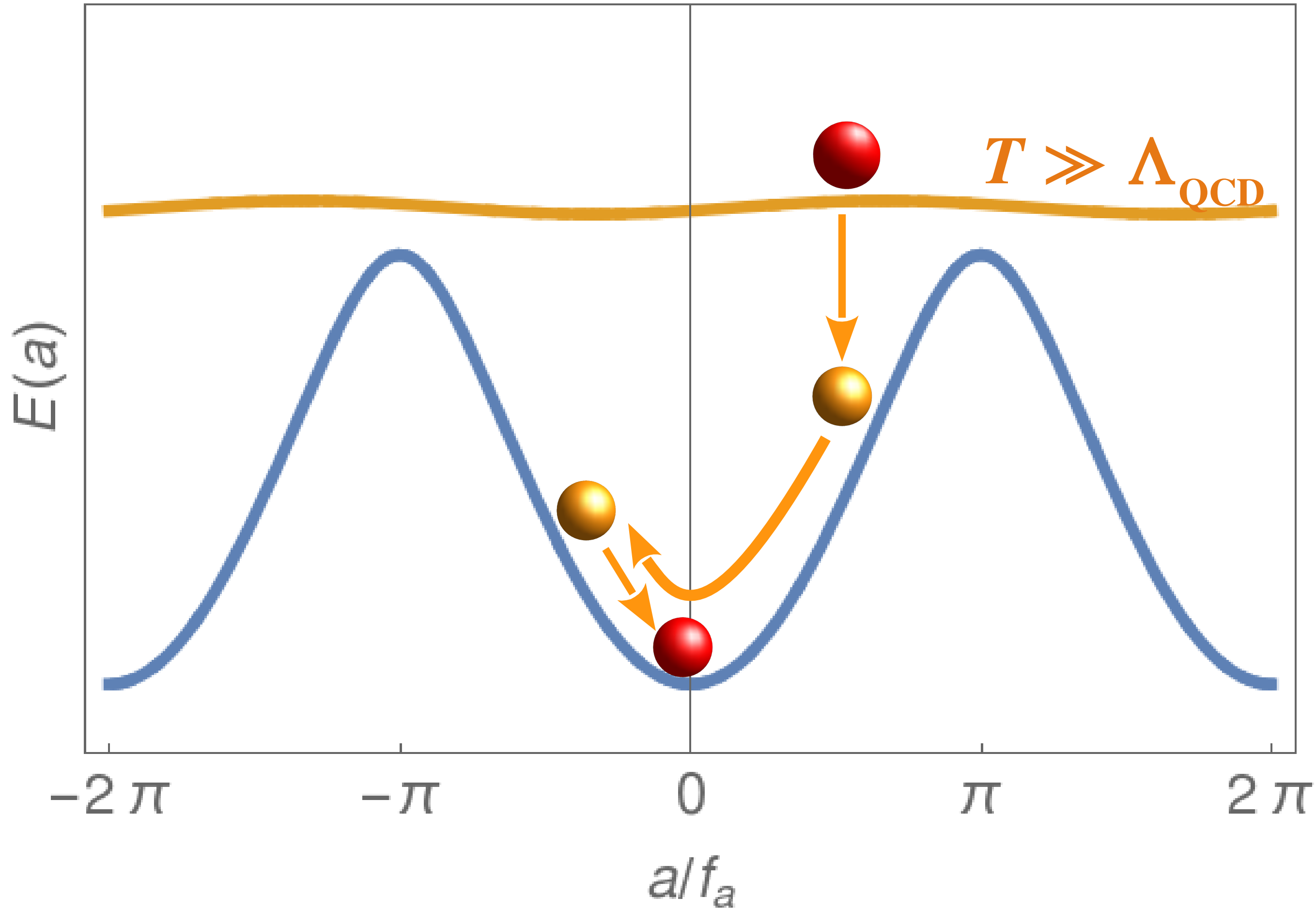
$$m_a \approx \frac{\Lambda^2_{\rm QCD}}{f_a}$$

2 birds with 1 stone

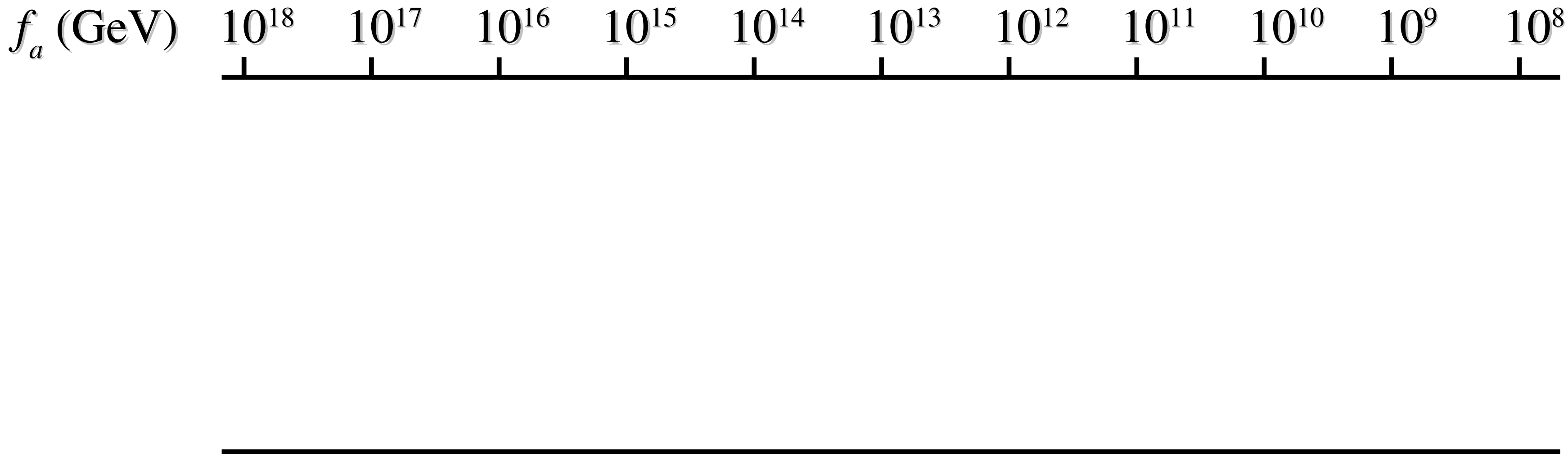
the QCD axion: *as dark matter*



the QCD axion: *as dark matter*



axion hunting



f_a (GeV)
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
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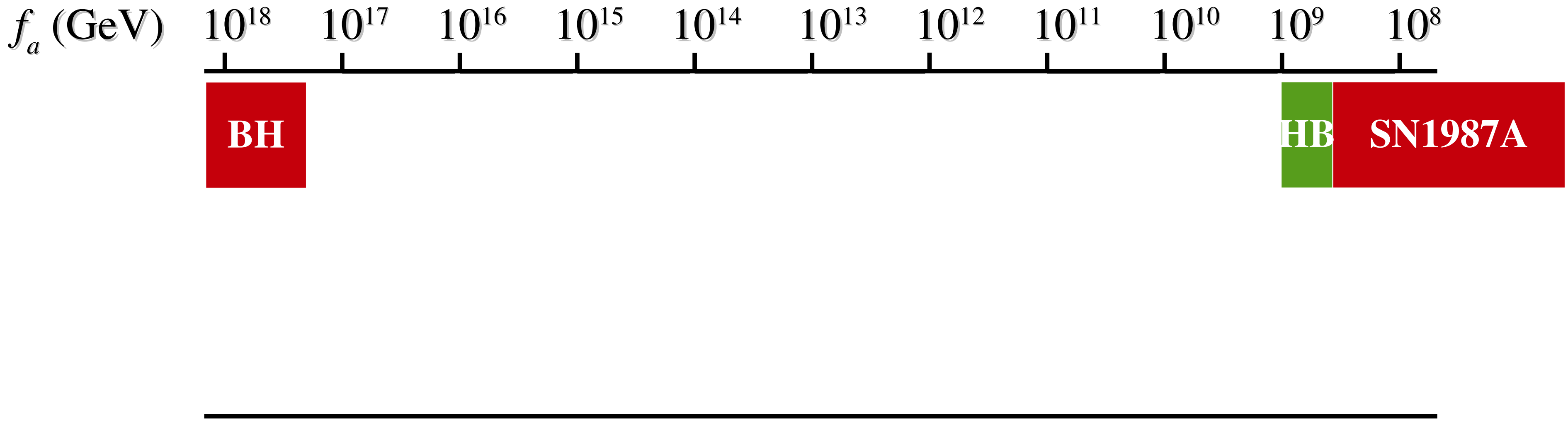
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}



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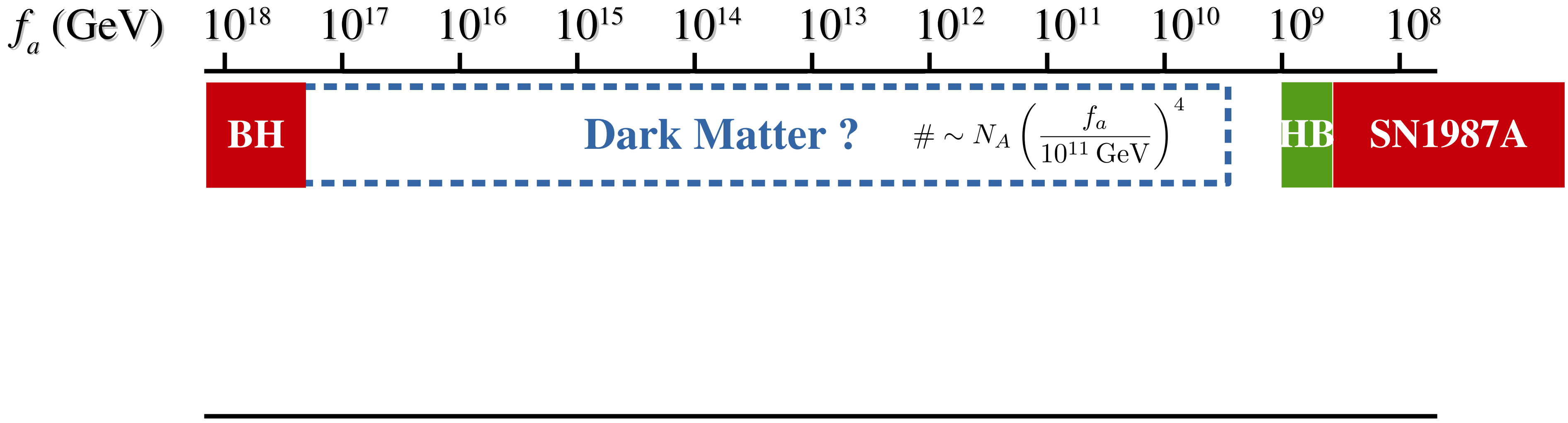
BH
HB
SN1987A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Dark Matter ?
$\# \sim N_A \left(\frac{f_a}{10^{11}\,\mathrm{GeV}}\right)^4$
HB
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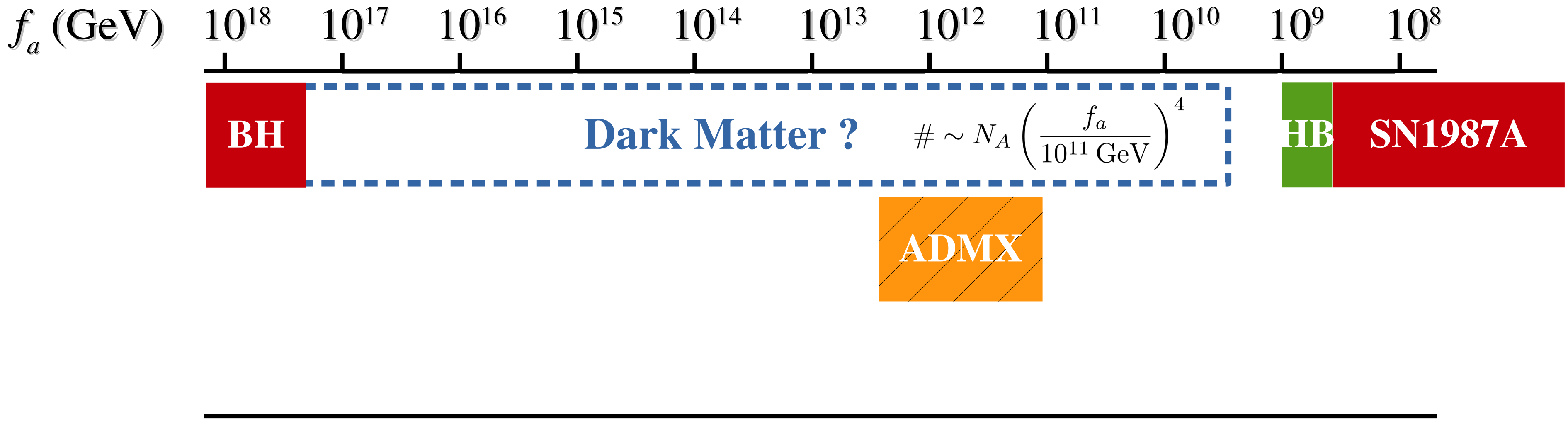
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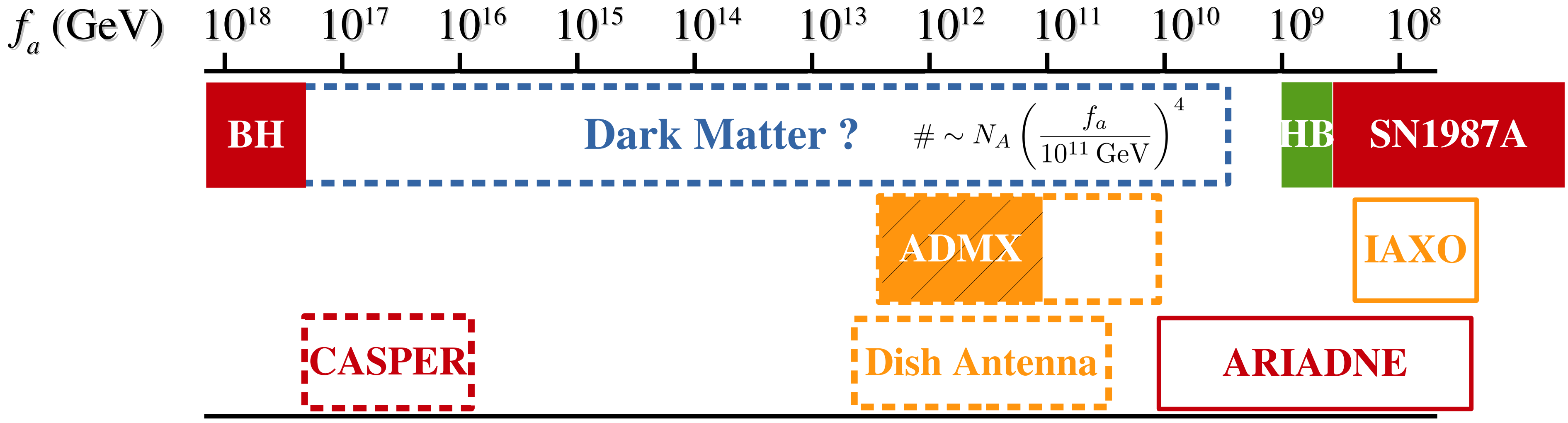

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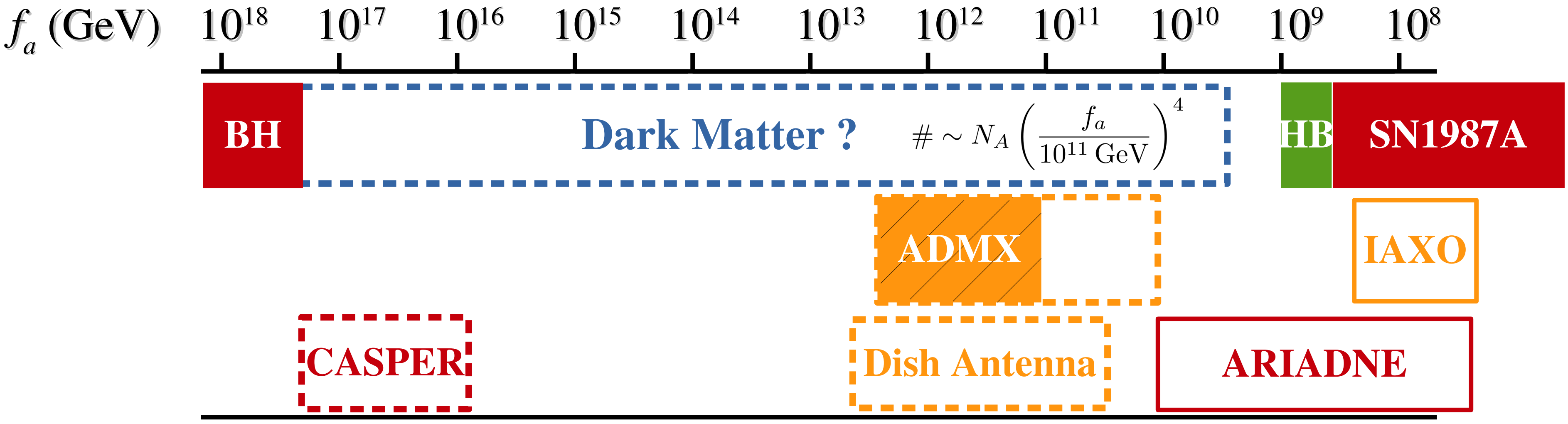
HB
SN1987A
ADMX
IAXO
CASPER
Dish Antenna
ARIADNE



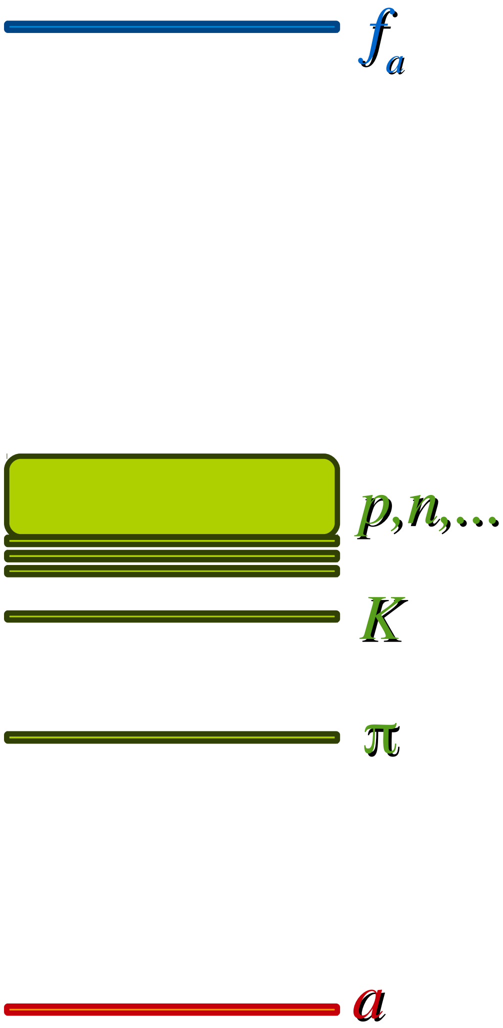
Axion DM ⇨ $\delta a/f_a \sim 10^{-19}$

Resonant Exp. ⇨ $\delta m/m \sim 10^{-6}$

Other couplings?

QCD axion properties

the QCD axion: *and its EFT*



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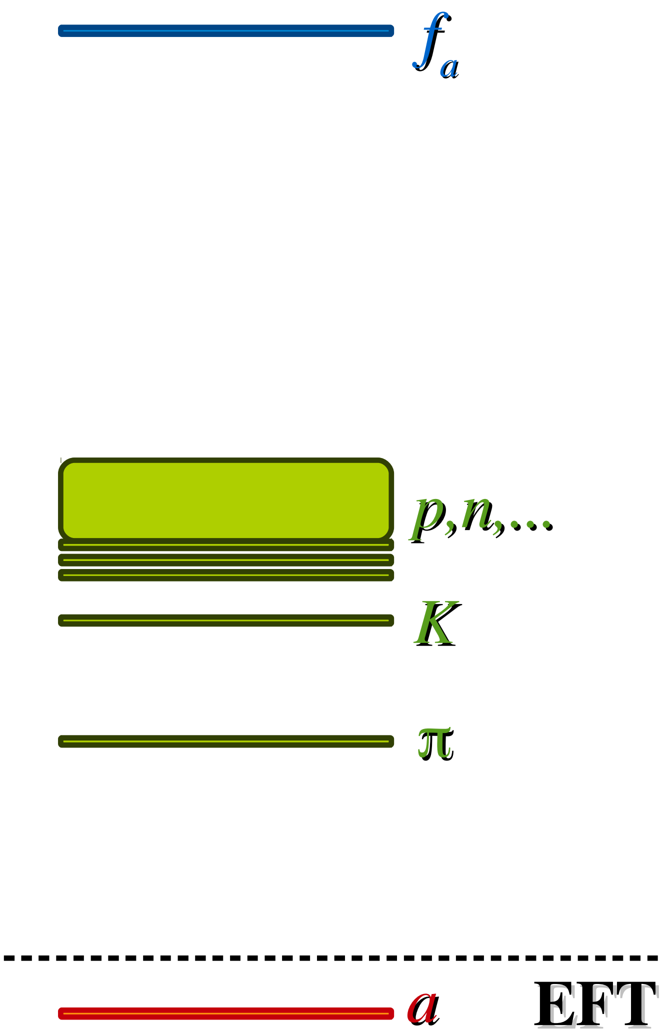

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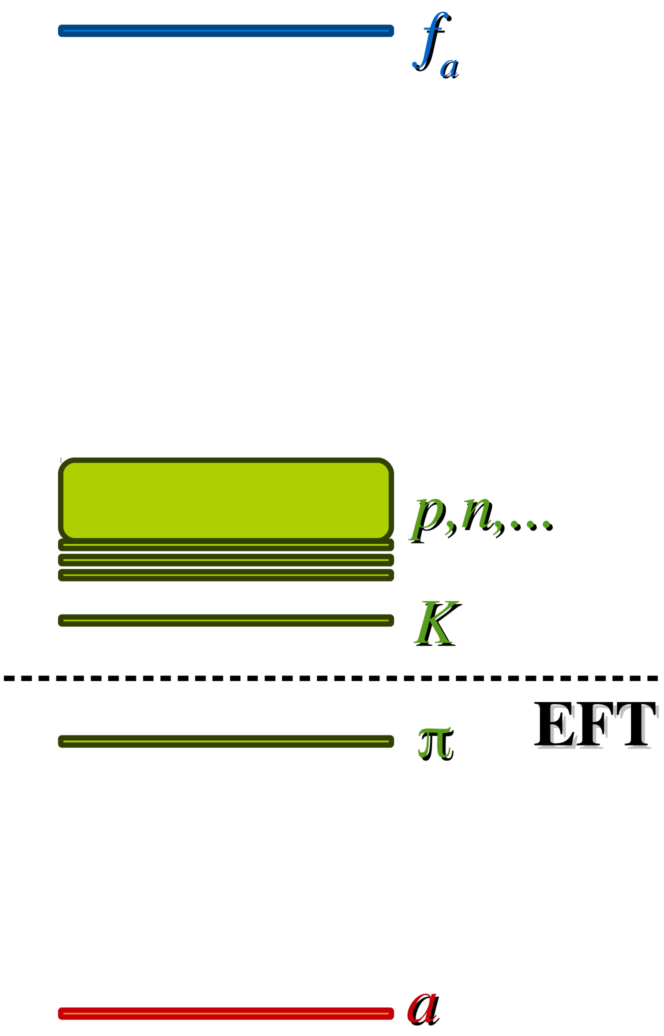

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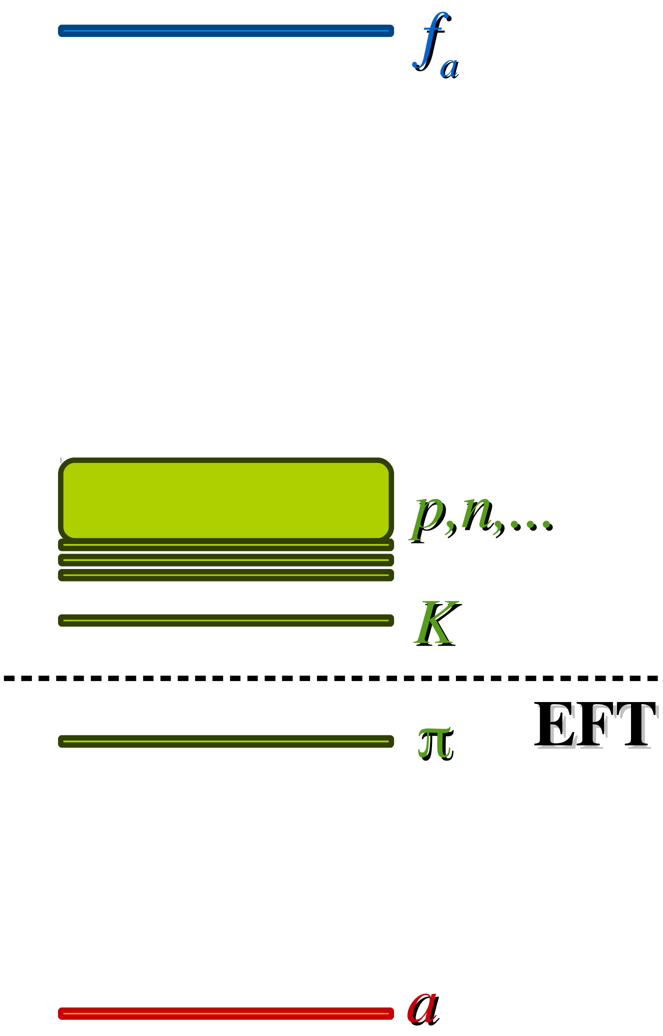

$$\frac{a}{32\pi^2 f_a} G\tilde{G} \;\rightarrow\; \overline{q}_L M_q e^{ia/2f_a} q_R + h.c.$$

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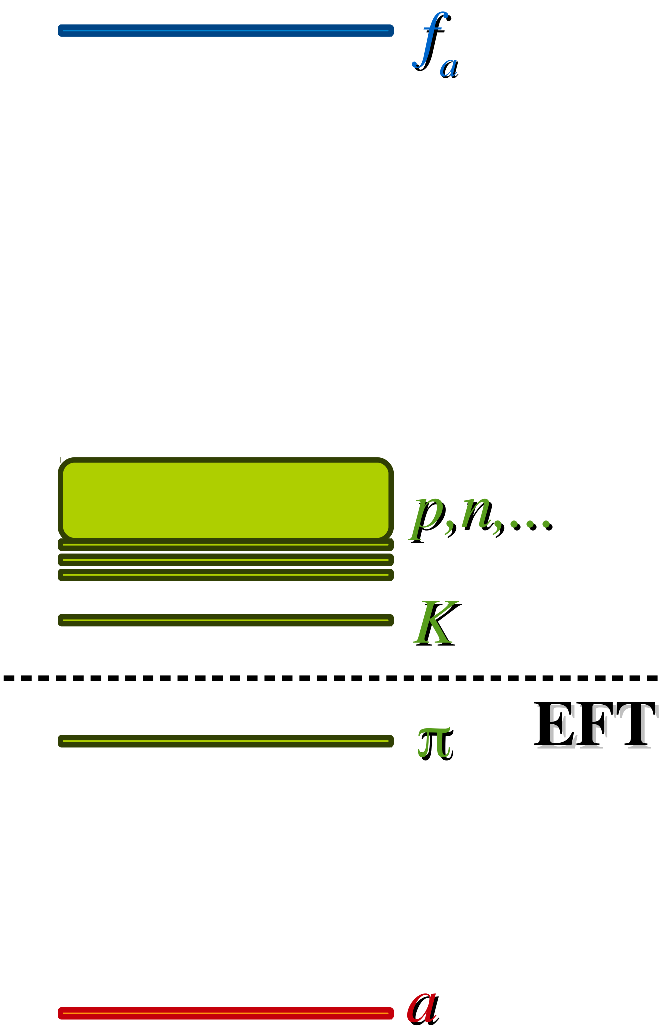

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$$\boxed{\mathcal{L}_{\rm QCD}(A_\mu, M_q e^{ia/2f_a}) \rightarrow \mathcal{L}_{\rm ChPT}(A_\mu, M_q e^{ia/2f_a})}$$

$$A_\mu = \partial_\mu a / f_a$$

the axion is an external source
in the EFT at LO in $1/f_a$

the QCD axion: *potential*

$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

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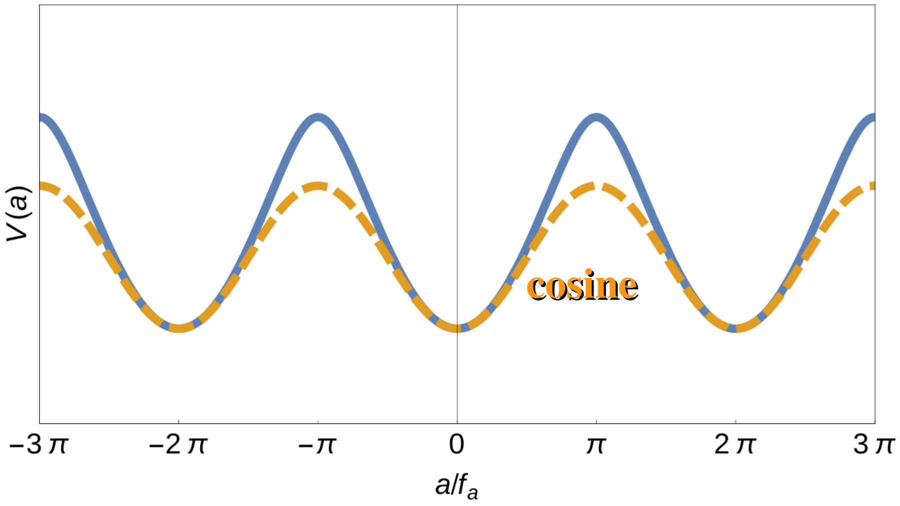
$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$

Di Vecchia Veneziano '80



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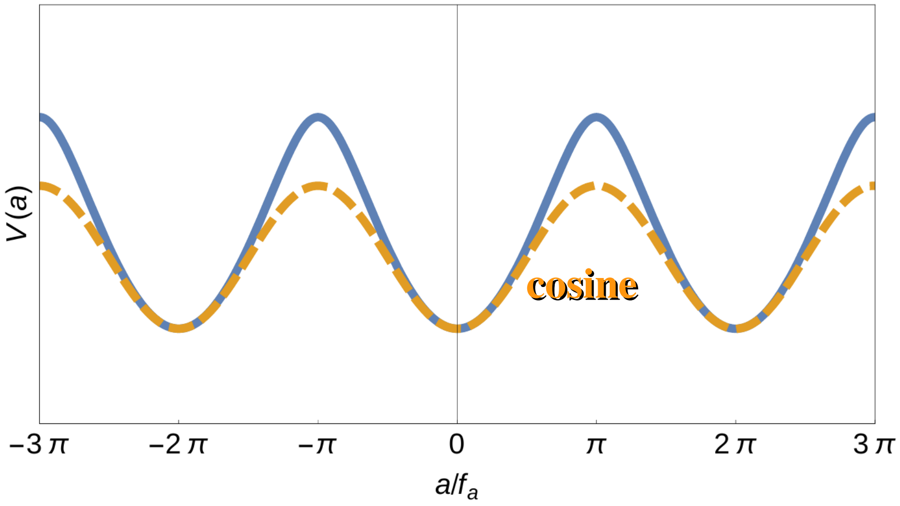

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

Weinberg '78

the QCD axion: *the mass @NLO*

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$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} \left[1 + 2 \frac{m_\pi^2}{f_\pi^2} \left(h_1^r - h_3^r - l_4^r + \frac{m_u^2 - 6 m_u m_d + m_d^2}{(m_u + m_d)^2} l_7^r \right) \right]$$

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}\left[1 + 2\frac{m_\pi^2}{f_\pi^2}\left(\underbrace{h_1^r - h_3^r - l_4^r}_{4.8 \pm 1.4 \cdot 10^{-3}} + \frac{m_u^2 - 6 m_u m_d + m_d^2}{(m_u+m_d)^2}\underbrace{l_7^r}_{7(4) \cdot 10^{-3}}\right)\right]$$

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$h_1^r - h_3^r - l_4^r$: $4.8 \pm 1.4 \cdot 10^{-3}$

l_7^r: $7(4) \cdot 10^{-3}$

lattice average:

$$z \equiv \frac{m_u^{\overline{\rm MS}}(2~\text{GeV})}{m_d^{\overline{\rm MS}}(2~\text{GeV})} = 0.48(3)$$

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$7(4) \cdot 10^{-3}$

lattice average:

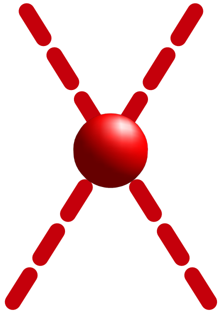
$$z \equiv \frac{m_u^{\overline{\rm MS}}(2\ {\rm GeV})}{m_d^{\overline{\rm MS}}(2\ {\rm GeV})} = 0.48(3)$$

$$m_a = 5.70(6)(4)\ \mu{\rm eV}\left(\frac{10^{12}{\rm GeV}}{f_a}\right)$$

$$(\chi^{top})^{1/4} = \sqrt{m_a f_a} = 75.5(5)\,{\rm MeV}$$

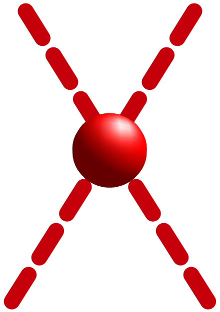
the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2} \quad \frac{m_u^2 - m_u m_d + m_d^2}{(m_u + m_d)^2}$$



the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2}\left\{\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} + 6\frac{m_\pi^2}{f_\pi^2}\frac{m_u m_d}{(m_u+m_d)^2}\left[h_1^r - h_3^r - l_4^r + \frac{4\bar{l}_4 - \bar{l}_3 - 3}{64\pi^2} - 4\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right]\right\}$$



$$\lambda_a = -0.346(22)\cdot\frac{m_a^2}{f_a^2}$$

the QCD axion: *potential @NLO*

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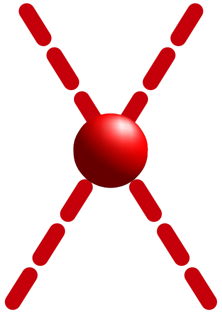
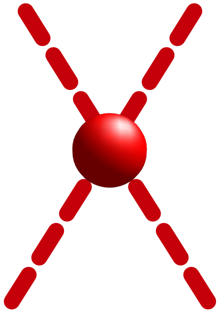


$$\lambda_a = -0.346(22)\cdot\frac{m_a^2}{f_a^2}$$

cosine→1

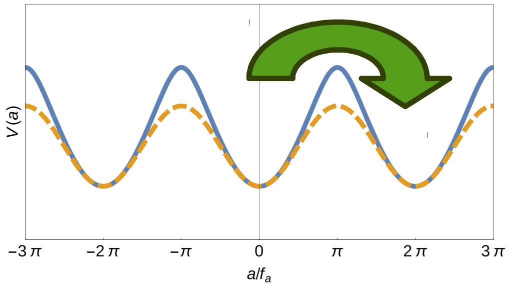
the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2}\left\{\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2}\right.$$
$$\left. +6\frac{m_\pi^2}{f_\pi^2}\frac{m_u m_d}{(m_u+m_d)^2}\left[h_1^r - h_3^r - l_4^r + \frac{4\bar{l}_4 - \bar{l}_3 - 3}{64\pi^2} - 4\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right]\right\}$$



$$\lambda_a = -0.346(22) \cdot \frac{m_a^2}{f_a^2}$$

cosine→1



domain wall

$$\sigma = 2 f_a \int_0^\pi d\theta \sqrt{2[V(\theta) - V(0)]} = 8.97(5)\, m_a f_a^2$$

the QCD axion: *potential @NLO*

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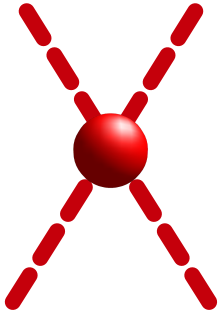

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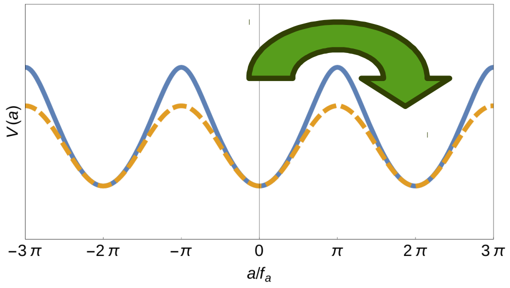

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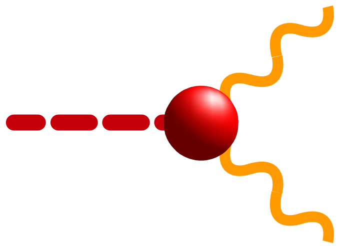
$$\sigma = 2f_a\int_0^\pi d\theta\,\sqrt{2[V(\theta)-V(0)]} = 8.97(5)\,m_a f_a^2$$

cosine→8

the QCD axion: *photon coupling @NLO*

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u}\right\}$$



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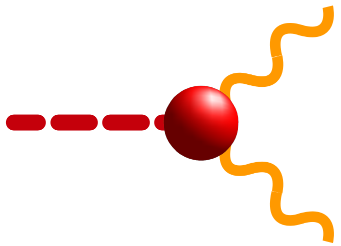

***E/N* =**

0	**(KSVZ,...)**
8/3	**(DFSZ, GUT-KSVZ,...)**
2	**(Unificaxion,...)**

the QCD axion: *photon coupling @NLO*

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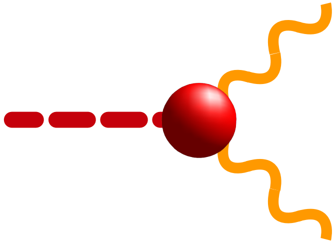

***E/N* =**

0 (KSVZ,...)

8/3 (DFSZ, GUT-KSVZ,...)

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tree ~ - 2

$a \to \pi \to \gamma\gamma$

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NLO $= 0.033(6)$

from $\pi \to \gamma\gamma$ $\eta \to \gamma\gamma$

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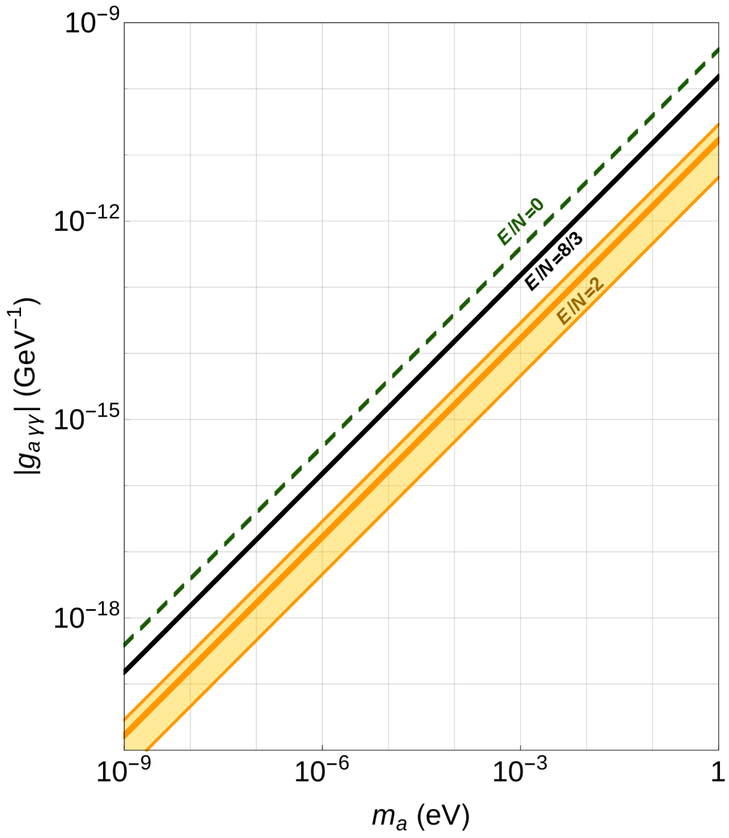
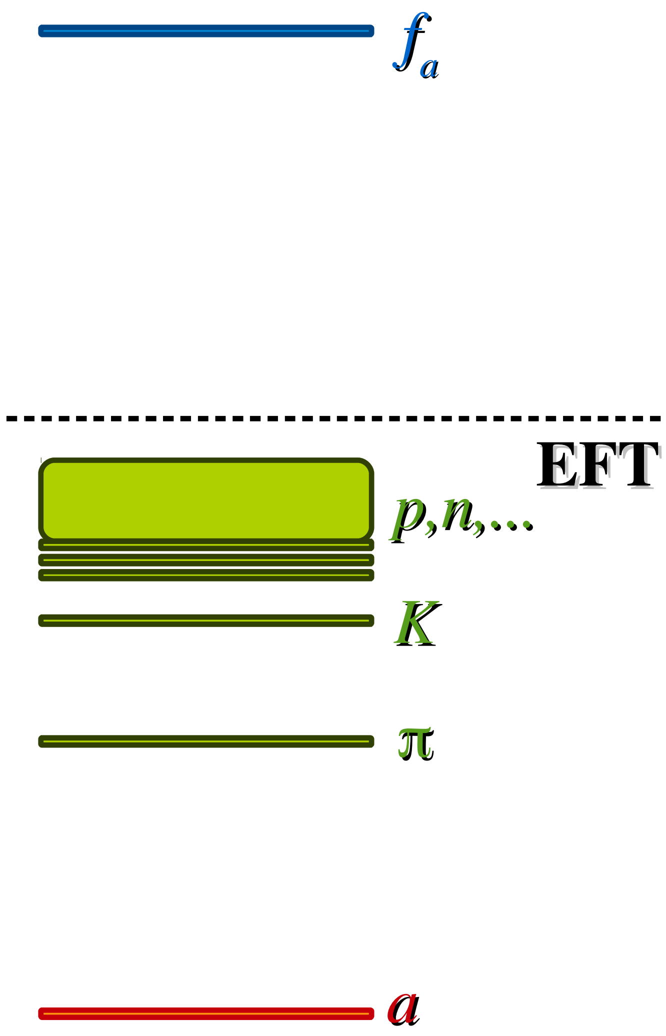
$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]$$

$$g_{a\gamma\gamma} = \begin{cases} -2.227(44)\cdot 10^{-3}/f_a & E/N = 0 \\ 0.870(44)\cdot 10^{-3}/f_a & E/N = 8/3 \\ 0.095(44)\cdot 10^{-3}/f_a & E/N = 2 \end{cases}$$

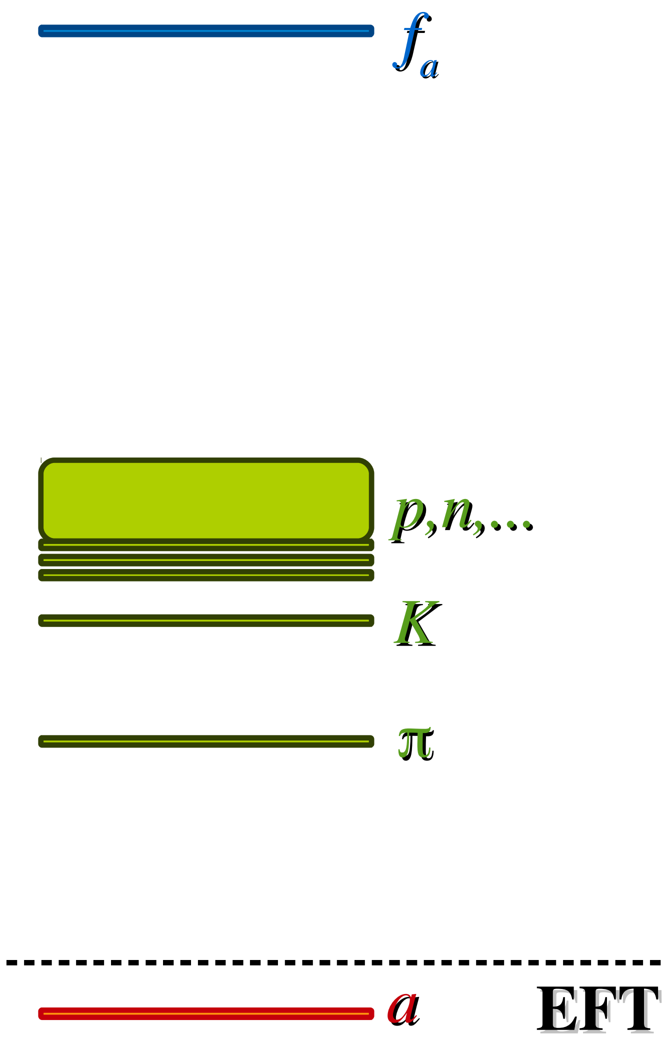


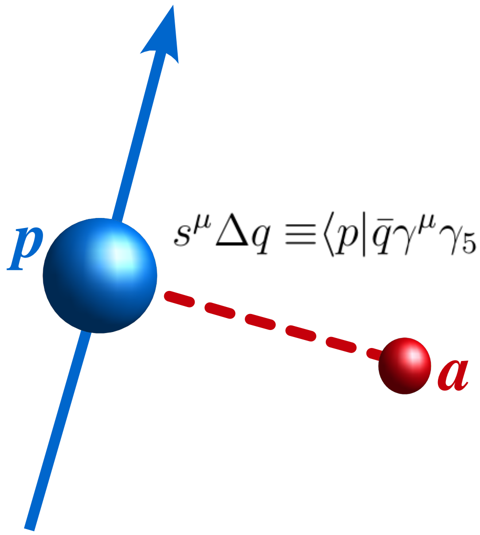
the QCD axion: *matter coupling*



the QCD axion: *matter coupling*

$$\mathcal{L}_N = \bar{N} v^\mu D_\mu N + 2 g_A \bar{N} S^\mu \hat{A}_\mu N + 2 g_0^i \bar{N} S^\mu N \bar{A}_\mu^i$$





the QCD axion: *matter coupling*

from β-decays: $\Delta u - \Delta d = g_A = 1.2723(23)$

from lattice QCD: $g_0^{ud} = \Delta u + \Delta d = 0.541(50)\,,\qquad \Delta s = -0.0227(34)\,,\qquad \Delta c = \pm 0.004$

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$$\boxed{\frac{\partial_\mu a}{2 f_a} c_N \bar{N} \gamma^\mu \gamma_5 N}$$

$$\begin{aligned} c_p &= -0.48(3) + 0.89(2) c_u^0 - 0.38(2) c_d^0 - 0.036(4) c_s^0 \\ &\qquad - 0.013(5) c_c^0 - 0.009(2) c_b^0 - 0.0036(4) c_t^0 \\ c_n &= -0.03(3) + 0.89(2) c_d^0 - 0.38(2) c_u^0 - 0.036(4) c_s^0 \\ &\qquad - 0.013(5) c_c^0 - 0.009(2) c_b^0 - 0.0036(4) c_t^0 \end{aligned}$$

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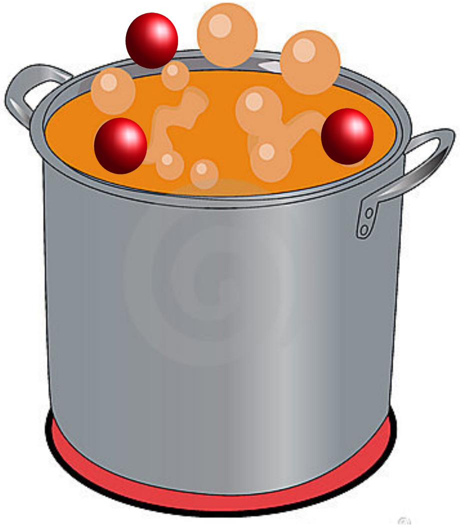
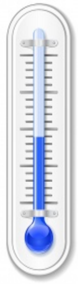
from RGE effects

model independent couplings



the *hot* axion

the QCD axion: *@ small temperature*



$$\frac{m_a^2(T)}{m_a^2} = 1 - \frac{3}{2}\frac{T^2}{f_\pi^2} J_1\left[\frac{m_\pi^2}{T^2}\right]$$

$$\frac{V(a;T)}{V(a)} = 1 + \frac{3}{2}\frac{T^4}{f_\pi^2 m_\pi^2\left(\frac{a}{f_a}\right)} J_0\left[\frac{m_\pi^2\left(\frac{a}{f_a}\right)}{T^2}\right]$$

$T < T_c \sim 155$ MeV

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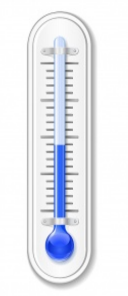

$$\frac{m_a^2(T)}{m_a^2} = 1 - \frac{3}{2}\frac{T^2}{f_\pi^2} J_1\left[\frac{m_\pi^2}{T^2}\right] \simeq 1 - \frac{3}{2(2\pi)^{3/2}}\frac{m_\pi^2}{f_\pi^2}\left[\frac{T}{m_\pi}\right]^{3/2} e^{-m_\pi/T}$$

$$\frac{V(a;T)}{V(a)} = 1 + \frac{3}{2}\frac{T^4}{f_\pi^2 m_\pi^2\left(\frac{a}{f_a}\right)} J_0\left[\frac{m_\pi^2\left(\frac{a}{f_a}\right)}{T^2}\right]$$

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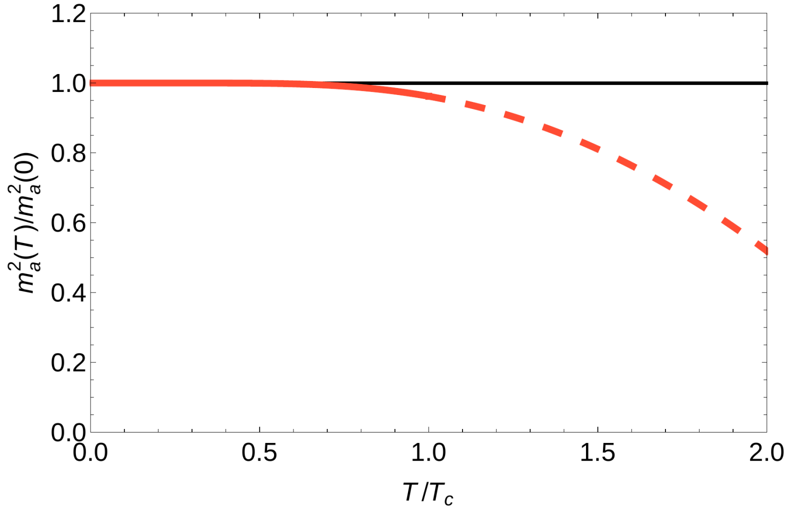
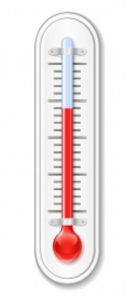


the QCD axion: *@ higher temperature*



T >> T_c

Gross Pisarski Yaffe '81

$$f_a^2 m_a^2(T) \simeq 2 \int d\rho\, n(\rho,0) e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \dots}$$

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integral over instanton sizes

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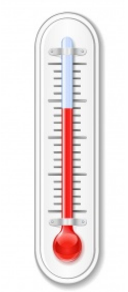

T >> T_c

$\propto m_u m_d e^{-8\pi^2/g_s^2(\rho)}$

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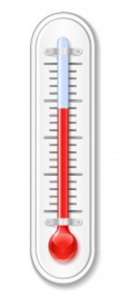

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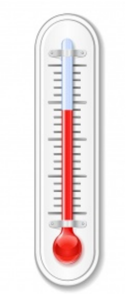
integral over instanton sizes

Debye screening cut-off from 1-loop thermal corrections

the QCD axion: *@ higher temperature*



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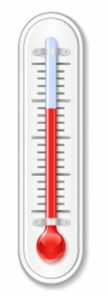

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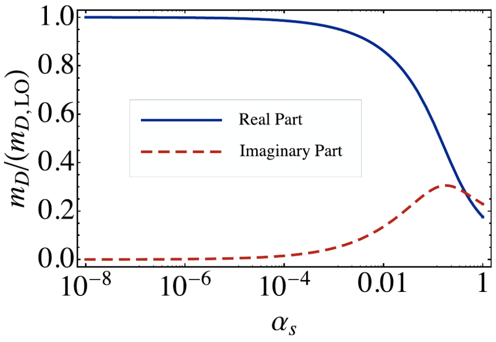
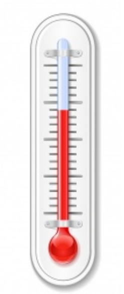


Bad convergence of thermal QCD
good only above T~$10^{5\text{-}6}$ GeV !!

the QCD axion: *@ higher temperature*



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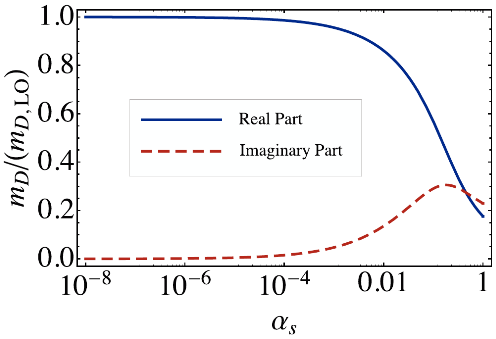

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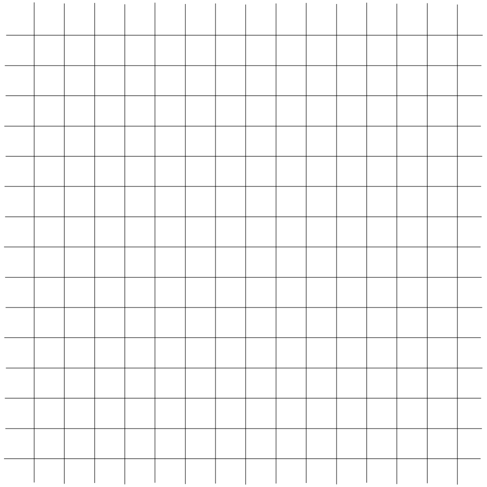
can we trust the instanton approx.?

the QCD axion from Lattice QCD

2+1 flavors with **physical** $\boldsymbol{m_q}$

$$\chi = \int d^4x \langle q(x) q(0) \rangle_{\theta=0} = \frac{\langle Q^2 \rangle_{\theta=0}}{\mathcal{V}}$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle^2_{\theta=0}}{12 \langle Q^2 \rangle_{\theta=0}}$$



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Temp. up to ~ 600 MeV (~ 4 T_c)

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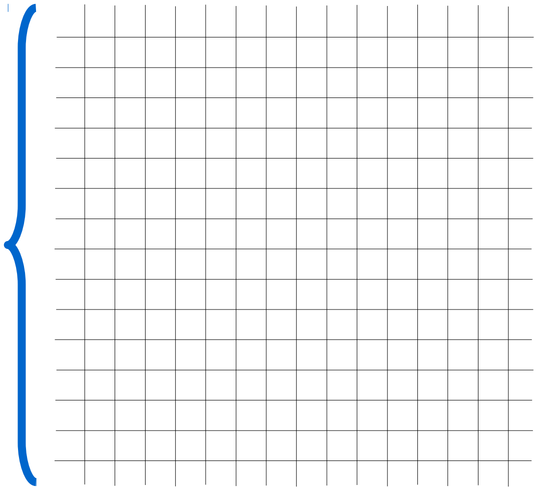
$48^3 \times 24 \div 48^3 \times 6$



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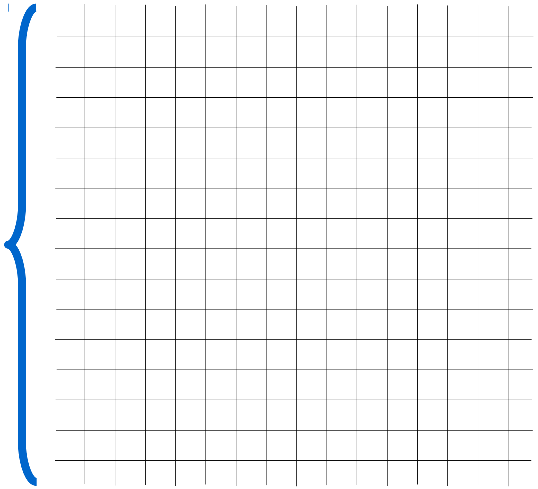
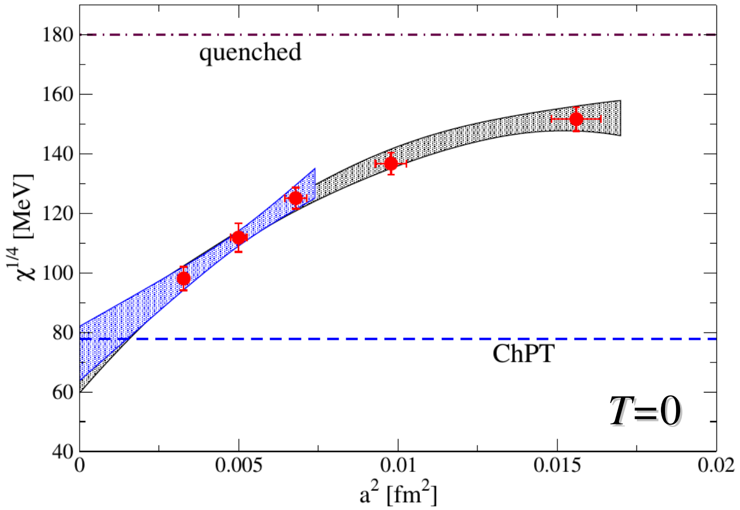


a = 0.082 ÷ 0.057 *fm*

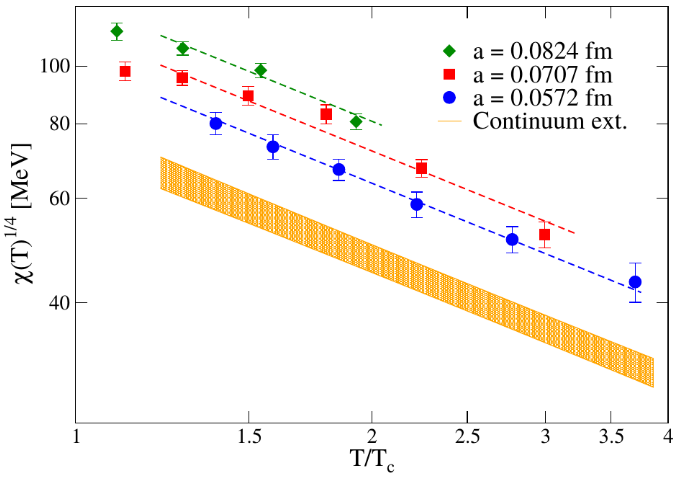
a^{-1} ~ 2.4 ÷ 3.5 GeV

Importance of continuum limit



the QCD axion from Lattice QCD

$$\chi = \int d^4x \langle q(x)q(0)\rangle_{\theta=0} = \frac{\langle Q^2\rangle_{\theta=0}}{\mathcal{V}} = m_a^2 f_a^2$$



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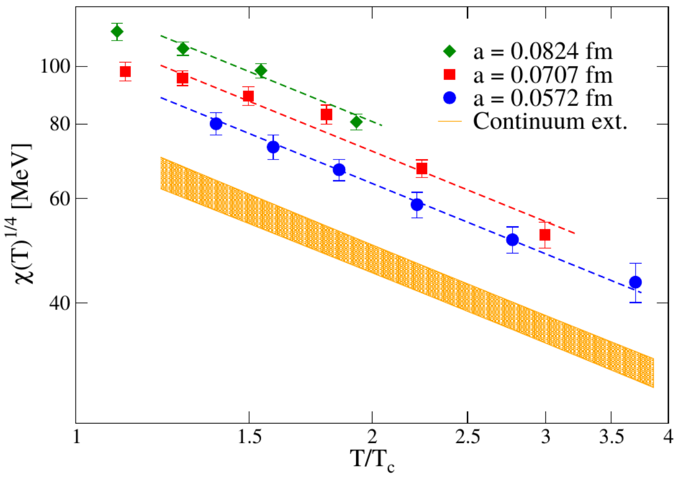
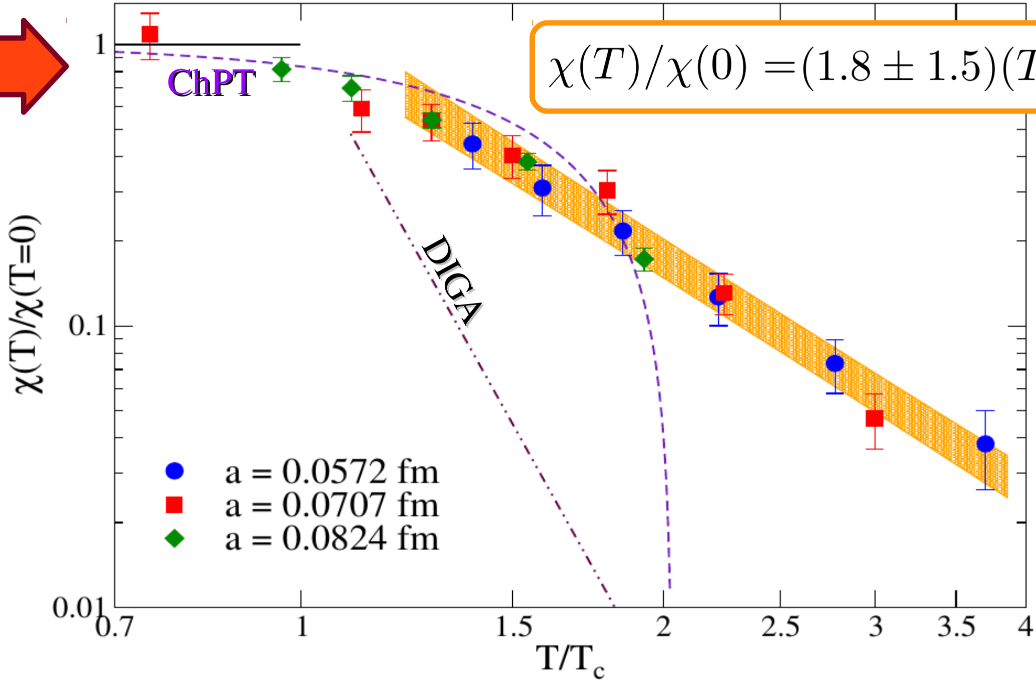
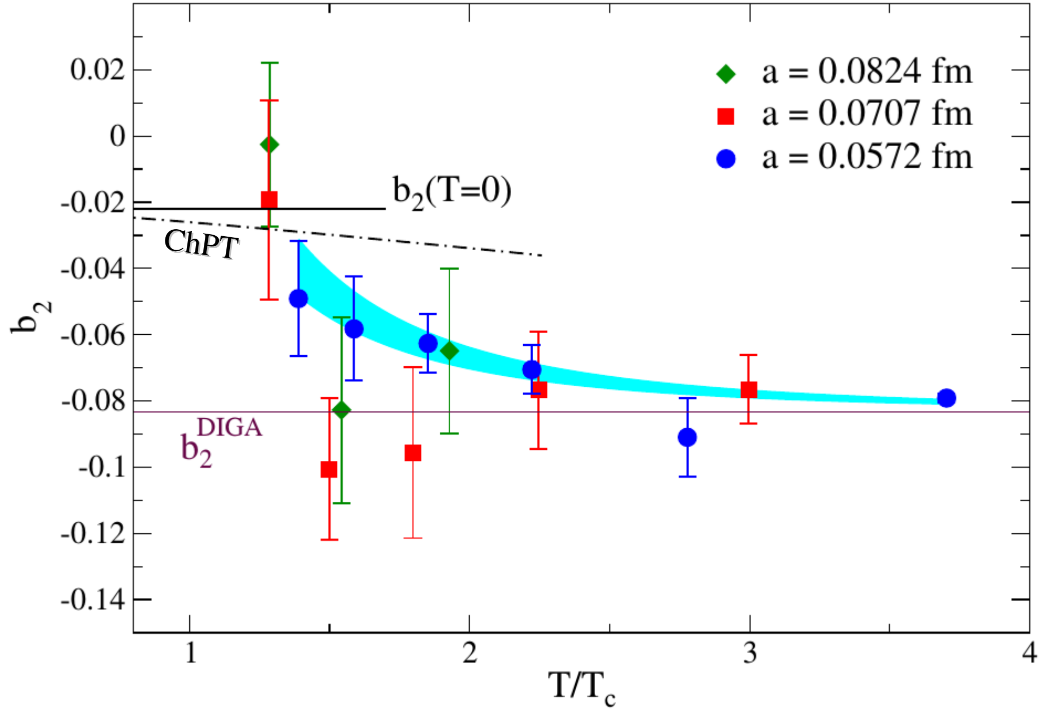




$\chi(T)/\chi(0) = (1.8 \pm 1.5)(T_c/T)^{2.90\pm0.65}$

the QCD axion from Lattice QCD

$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}} = \frac{\lambda_a}{12}\frac{f_a^2}{m_a^2}$$



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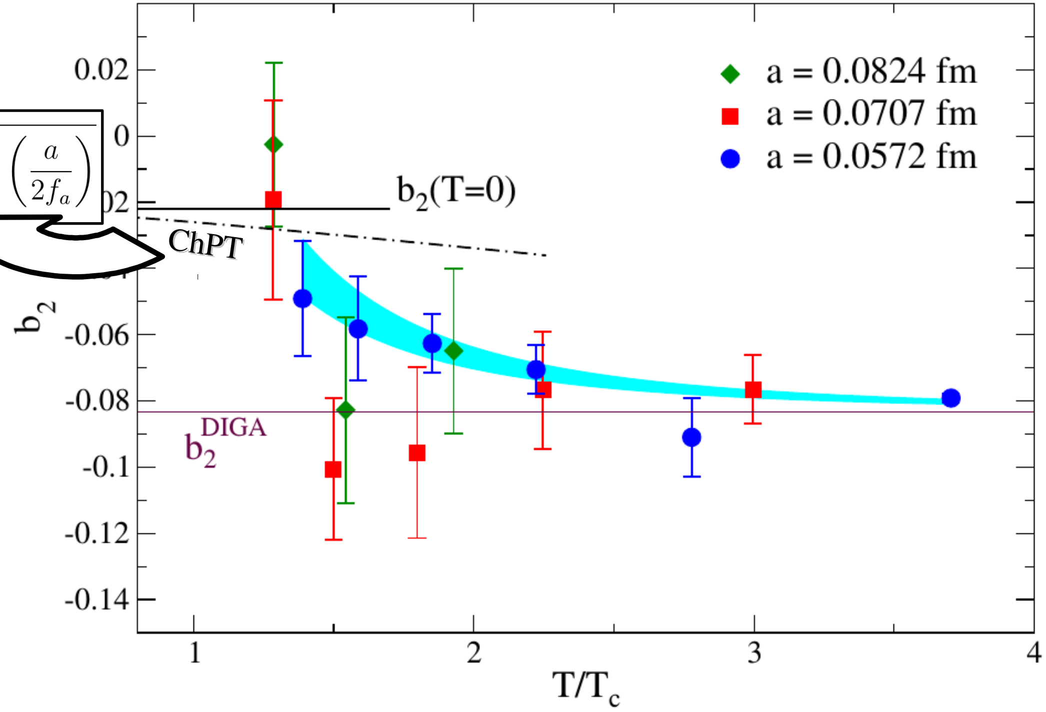
$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$



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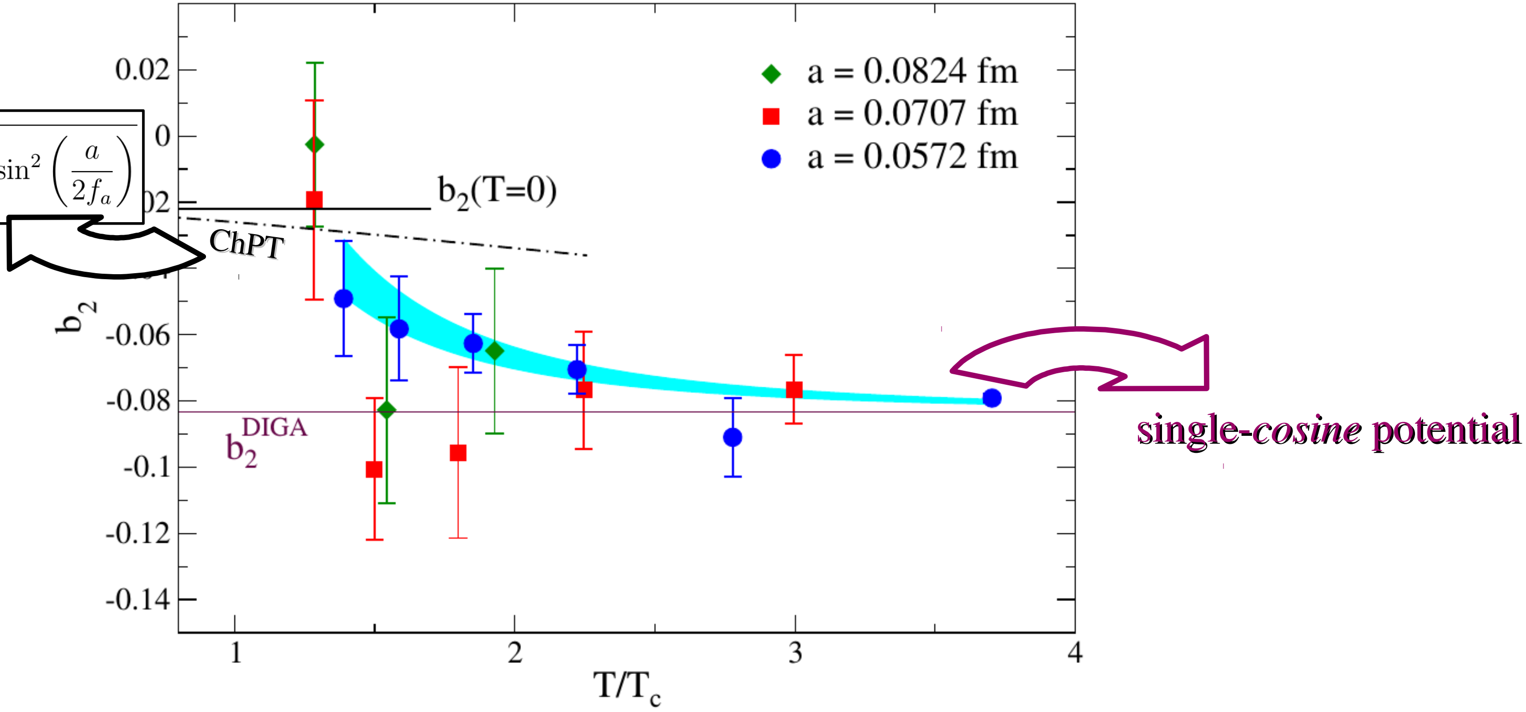
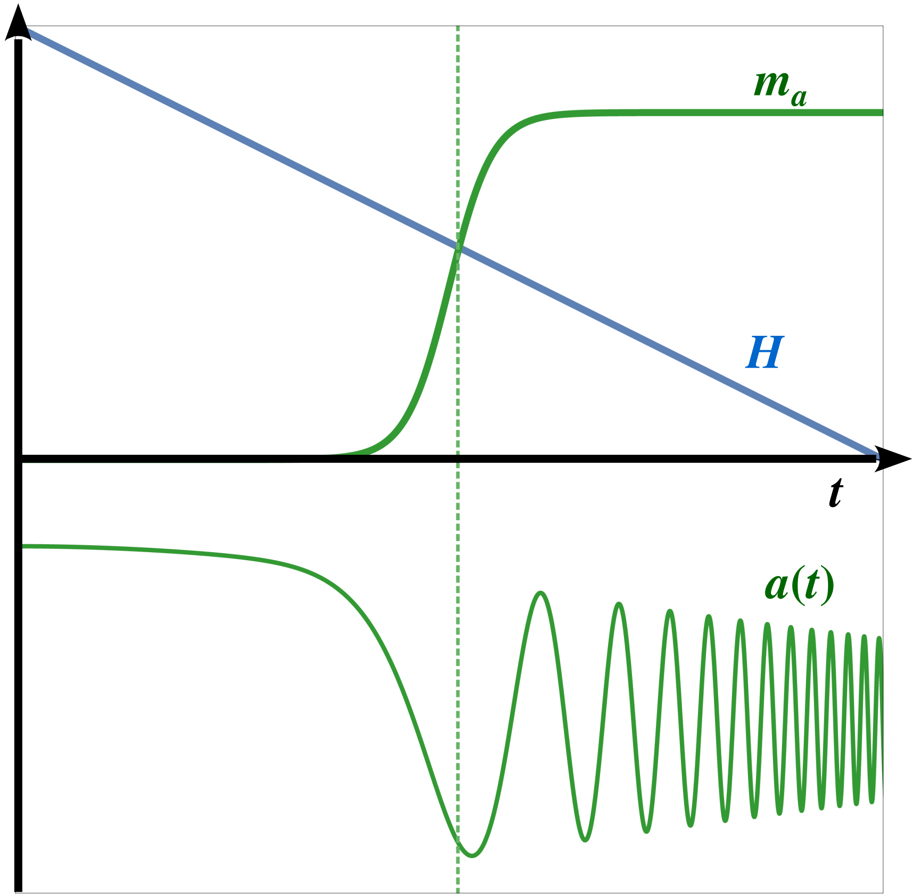


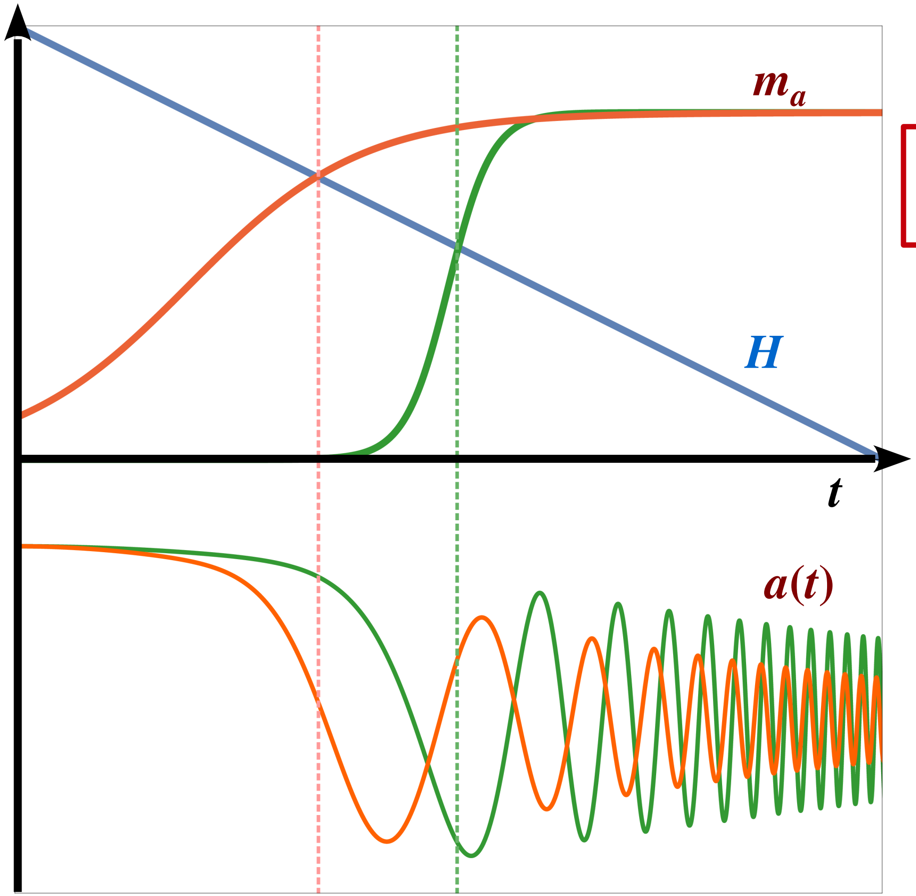
the QCD axion: *relic abundance*



$$\ddot{a} + 3H\dot{a} + m_a^2\,(T)\,f_a \sin\left(\frac{a}{f_a}\right) = 0$$

$$\rho_a = m_a^2 a^2$$

the QCD axion: *relic abundance*



$$m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha} = m_a^2\frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}$$

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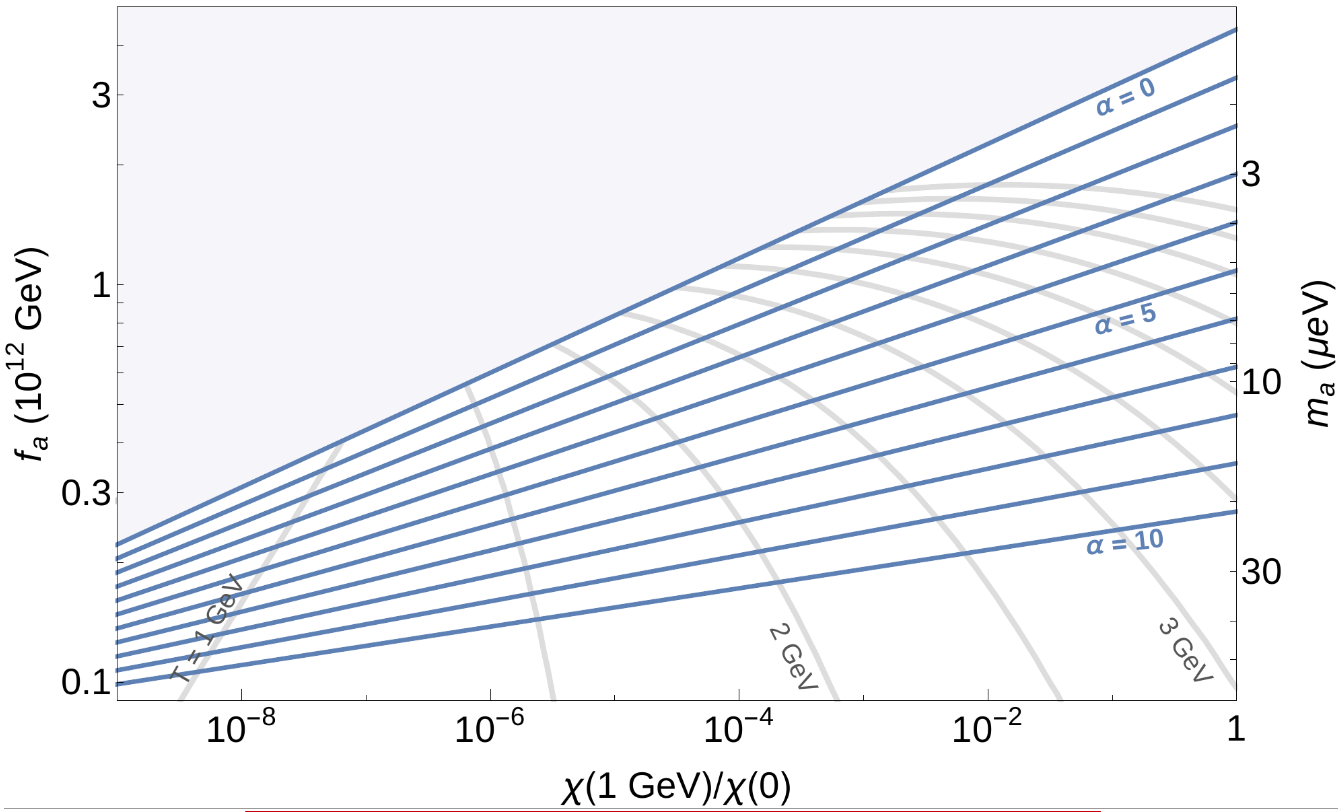

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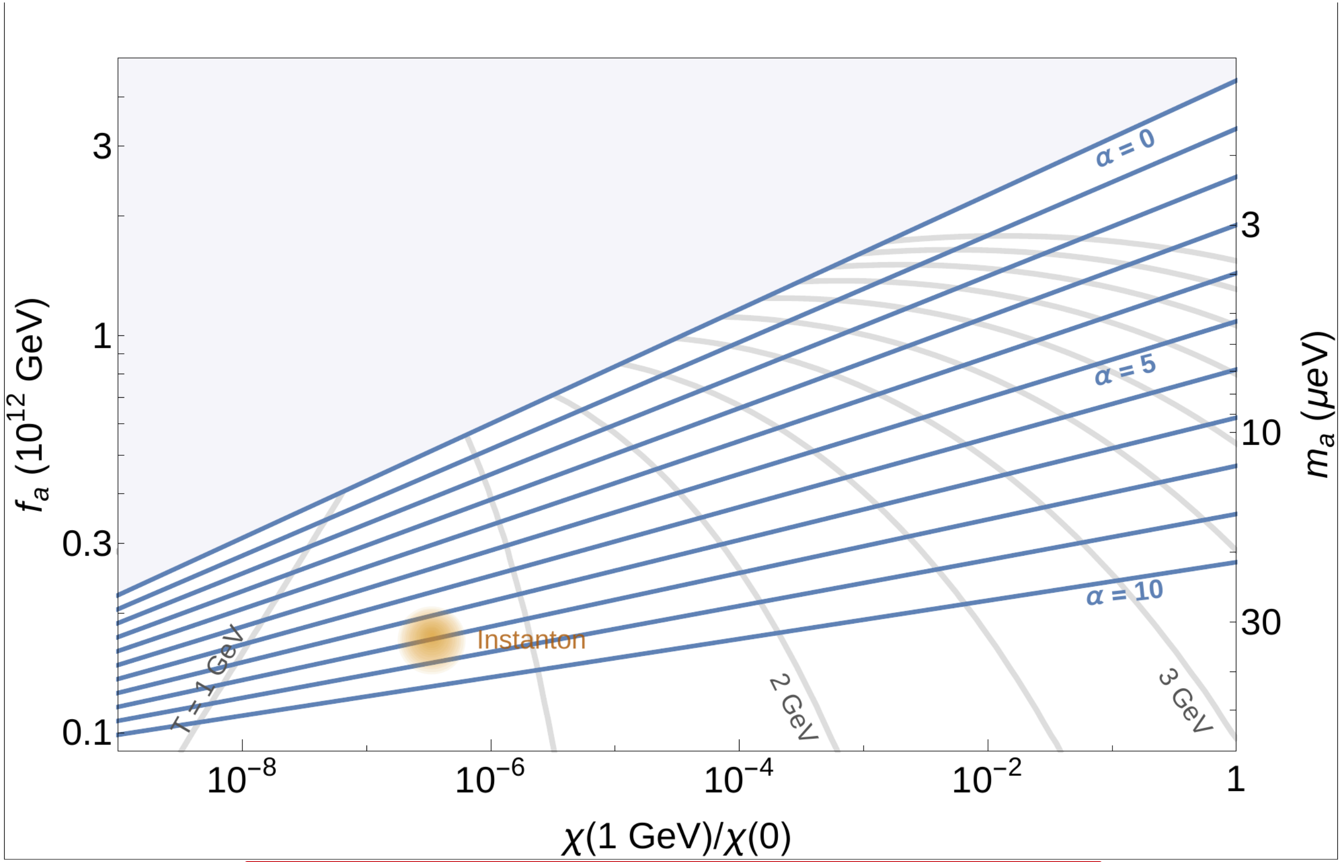

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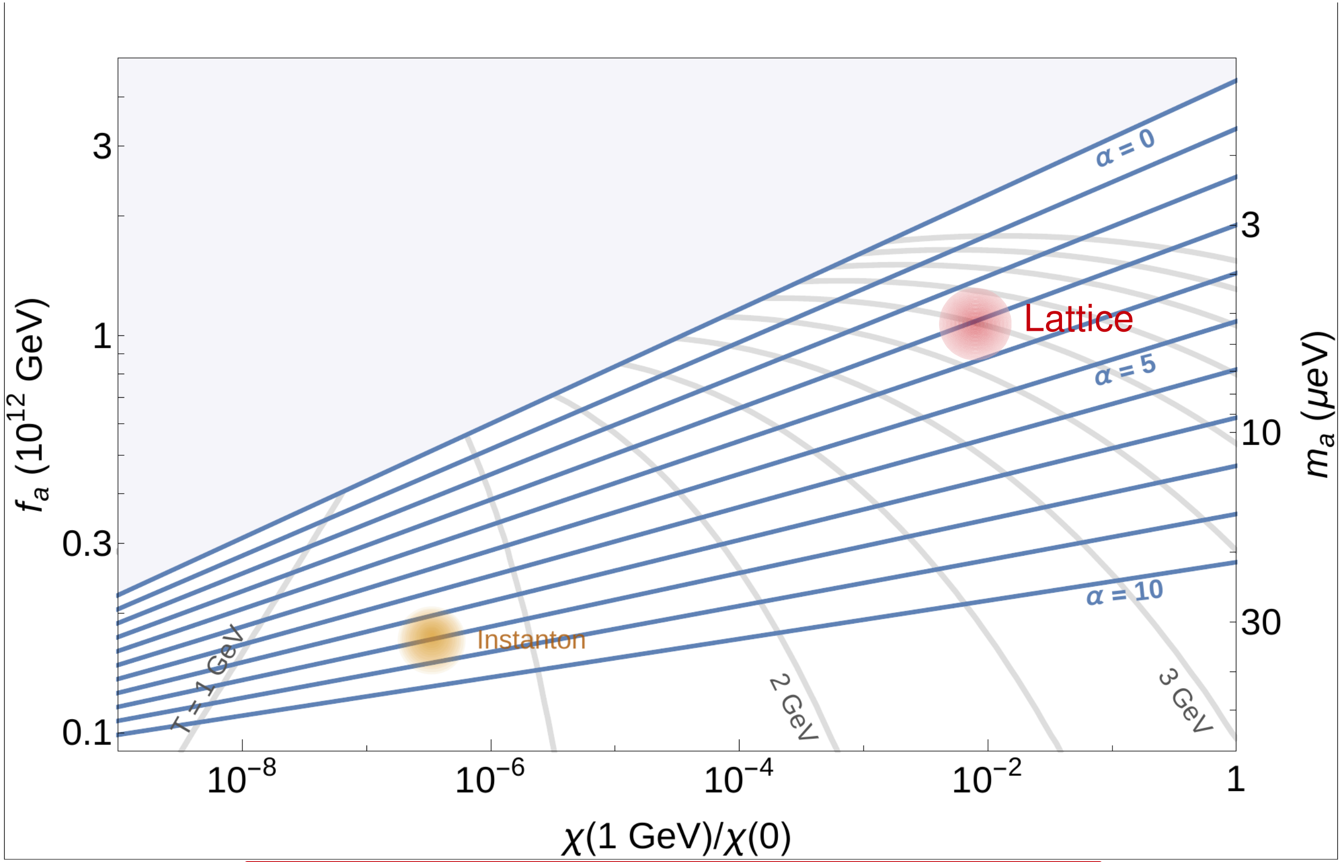

$$m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha} = m_a^2\frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}$$

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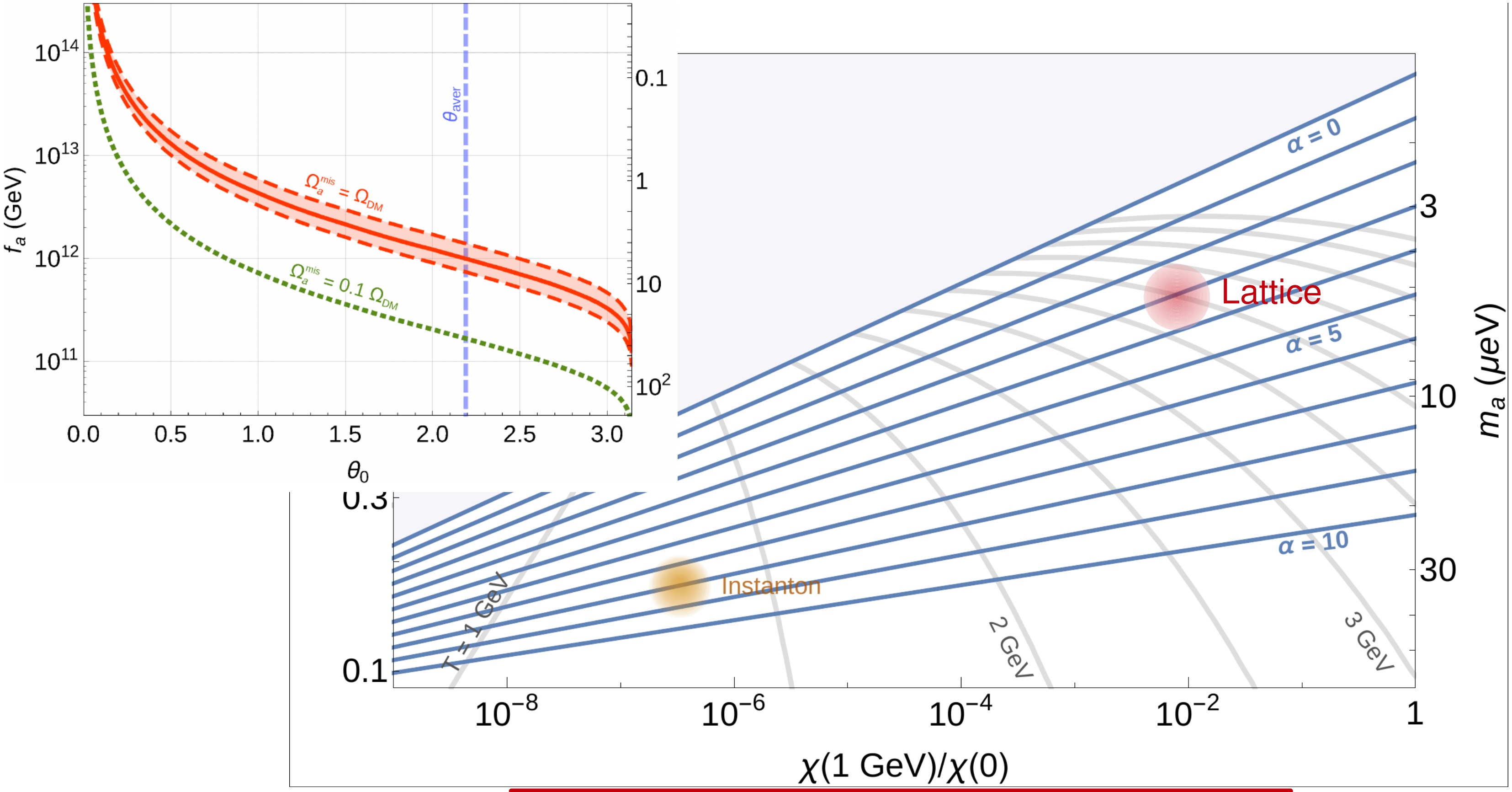

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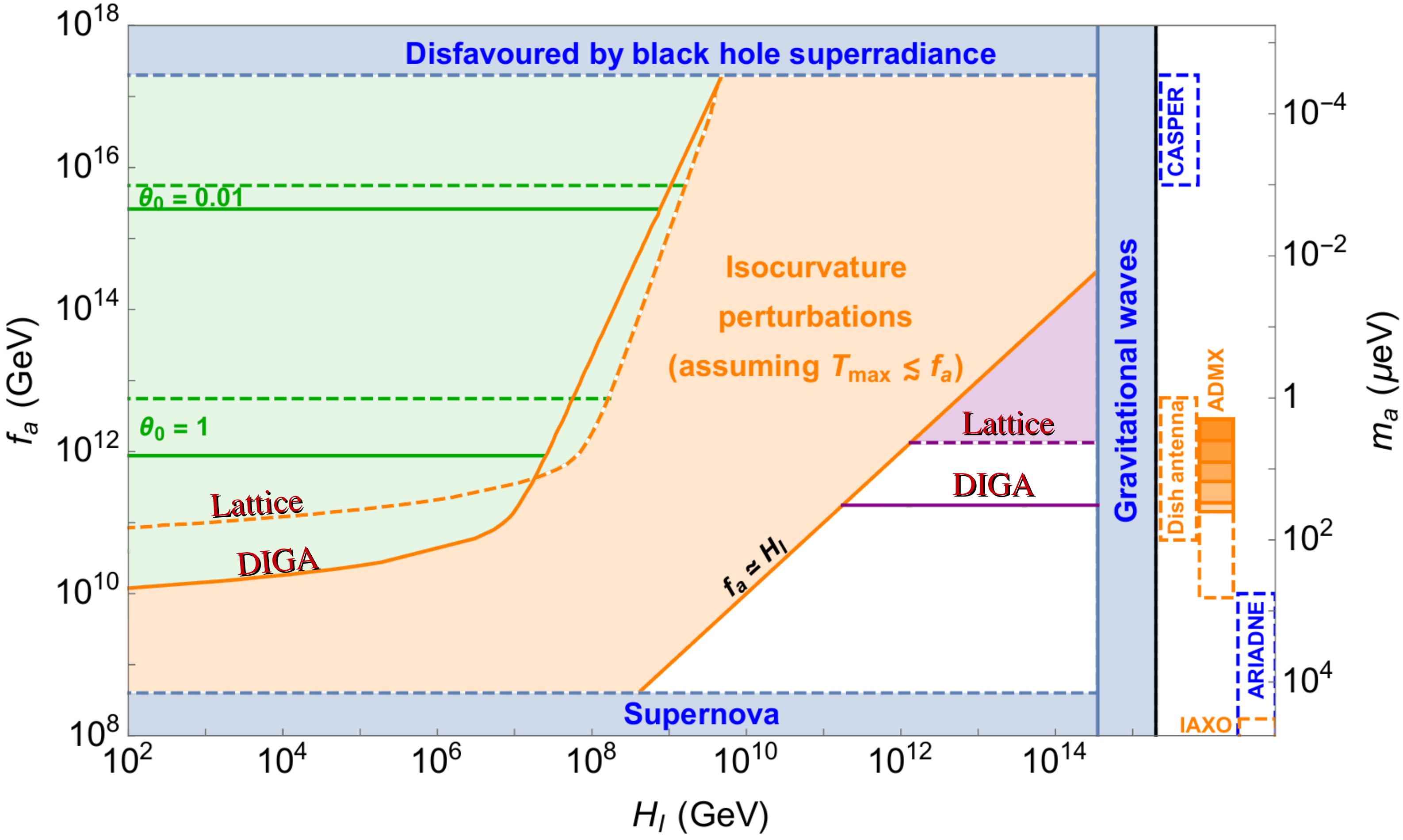
the QCD axion: *parameter space*



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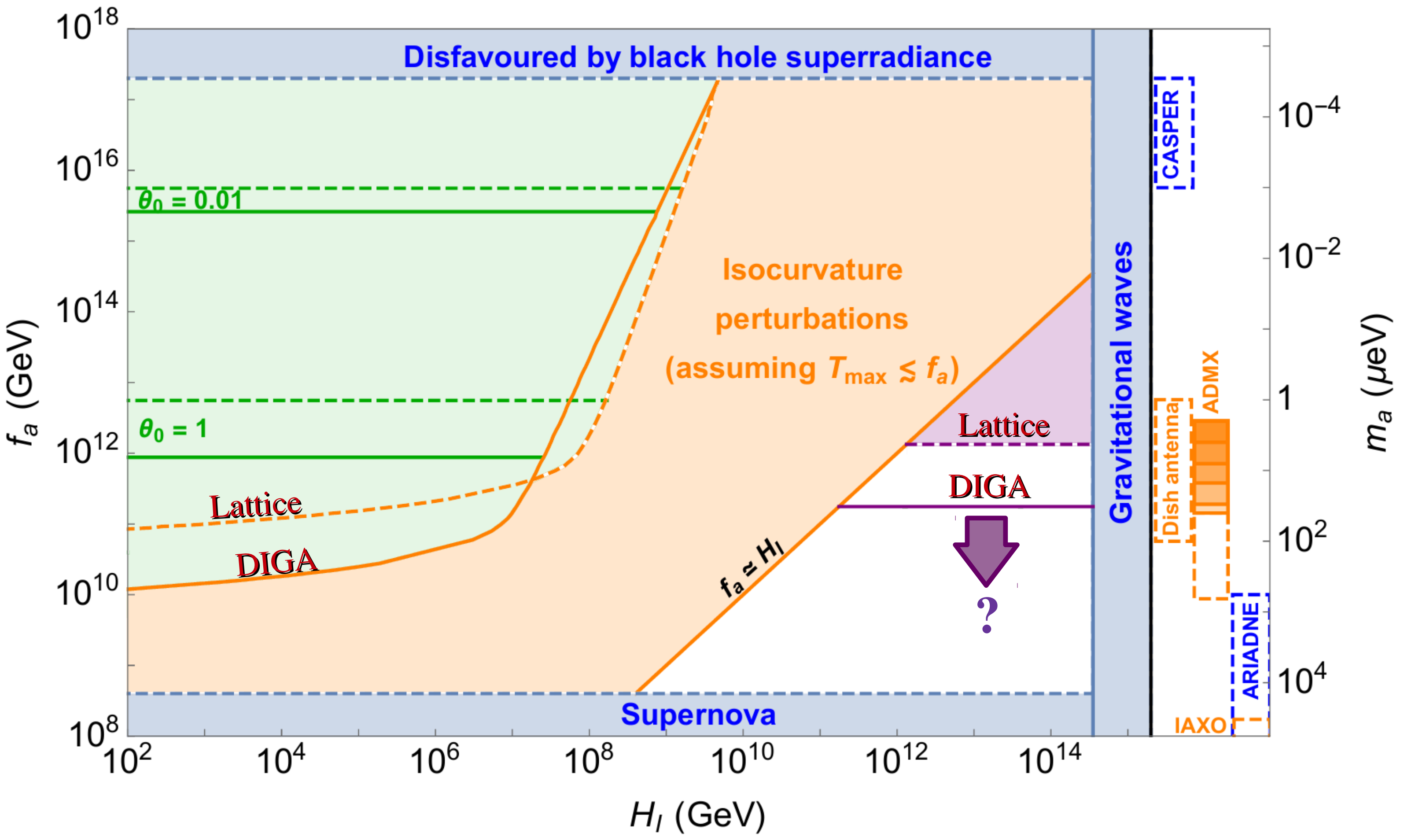


Conclusions:

Precision QCD axion physics:

already @ 1% - 10% accuracy
(room for improvement)

High temperature:

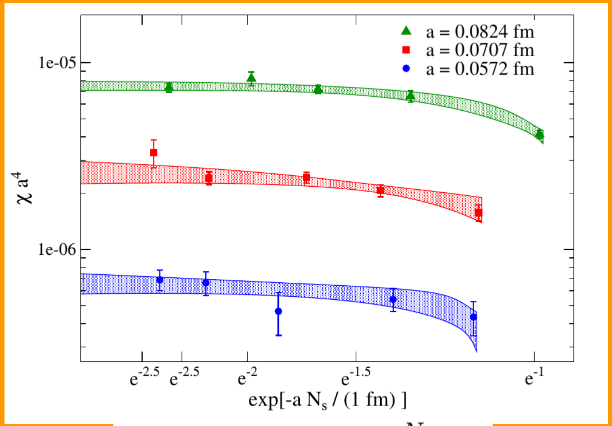
instantons unreliable → big deviations from lattice computation
further studies required

To Do:

- CP violating couplings
- relic abundance from topological defects?

Backup

Volume dependence

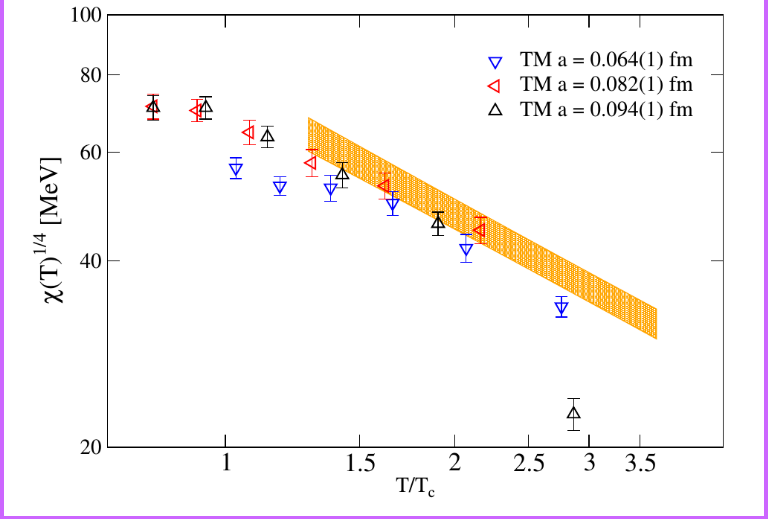


$$\chi_{N_s} \sim \chi_\infty + C e^{-a N_s m_{\eta'}}$$

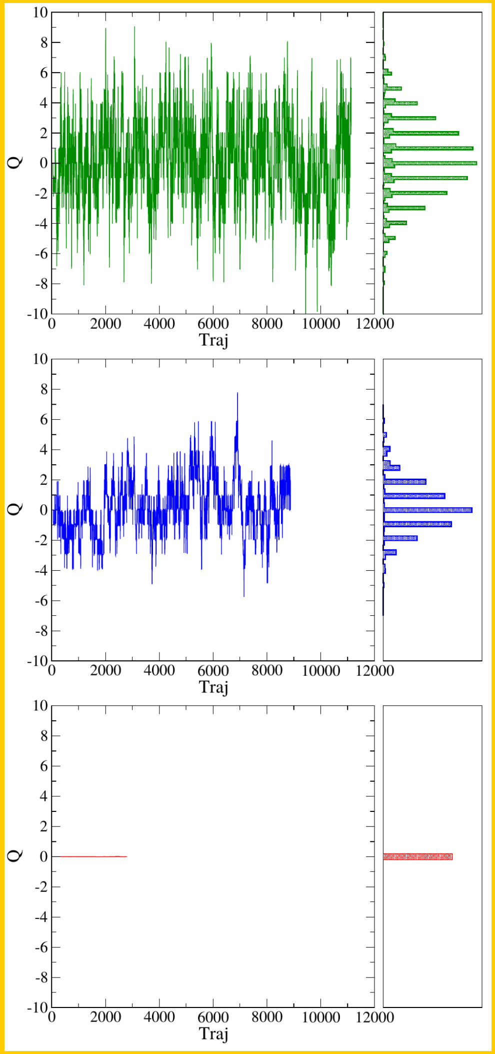
$$\chi = \int d^4x \langle q(x) q(0) \rangle_{\theta=0} = \frac{\langle Q^2 \rangle_{\theta=0}}{\mathcal{V}}$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3 \langle Q^2 \rangle^2_{\theta=0}}{12 \langle Q^2 \rangle_{\theta=0}}$$

Comparisons
with
Trunin et al. '15



freezing of topological charge



the QCD axion: *potential @ NLO*

$$V(a)^{\rm NLO} = -\, m_\pi^2\left(\tfrac{a}{f_a}\right) f_\pi^2 \left\{1 - 2\frac{m_\pi^2}{f_\pi^2}\left[l_3^r + l_4^r - \frac{(m_d - m_u)^2}{(m_d + m_u)^2} l_7^r - \frac{3}{64\pi^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)\right]\right.$$

$$\left. + \frac{m_\pi^2\left(\tfrac{a}{f_a}\right)}{f_\pi^2}\left[h_1^r - h_3^r + l_3^r + \frac{4m_u^2 m_d^2}{(m_u + m_d)^4}\frac{m_\pi^8 \sin^2\left(\tfrac{a}{f_a}\right)}{m_\pi^8\left(\tfrac{a}{f_a}\right)} l_7^r - \frac{3}{64\pi^2}\left(\log\left(\frac{m_\pi^2\left(\tfrac{a}{f_a}\right)}{\mu^2}\right) - \frac{1}{2}\right)\right]\right\}$$

$$m_\pi^2(\theta) = m_\pi^2\sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2}\sin^2\left(\frac{\theta}{2}\right)}$$

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